Financial Contracting when Rivals may Turn Nasty

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Abstract

This paper develops a principal–agent model of financial contracting in which optimal contracts resemble a combination of debt and equity. When defaulting on debt, the firm is punished by disruption of external funding. Such contracts however, invite rivals to compete more aggressively to increase the likelihood of default. The firm will respond to the threat of predation, by choosing a less leveraged capital structure, even though this will aggravate incentive problems. In contrast to the literature on debt as a device of strategic commitment, this result supports the common presumption that equity can enhance the firm's competitiveness in the product market.

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JEL class. G32, D82, D43

1 Introduction

According to the ‘long–purse’ story, cash–rich competitors may predate on financially constrained rivals in an attempt to drive them into bankruptcy and out of business. However, such a strategy only makes sense if capital markets do not perform well. With perfect capital markets, a firm would be able to raise funds for any worthwhile investment, even after suffering losses. Hence, predation would not pay. A comprehensive theory of predation, therefore, also has to offer an explanation of capital market imperfections. In this paper agency problems between the firm and its financiers provide the link between capital structure and competitive strength in the product market.

If free cash–flow is not contractible, optimal financial contracts, may punish the firm if their performance is poor e.g. by limiting investment, sacking the management or terminate funding and closing down its operations. While such sanctions deter the firm from appropriating cash–flow, they can also encourage hostility in the product market. If operations are suspended, a rival may predate in order to become a monopolist. If investment is curbed, competitors may gain market–share or cost–advantages by inflicting harm. As an alternative to punishing poor performance, financiers may curtail the firm’s control of resources and intensify monitoring to make diversion of free–cash–flow more difficult. A more rigid control regime increases the cost of appropriating cash–flow and allows for softer pay–incentives and less use of sanctions. It will, however, decrease the flexibility of operations and require higher monitoring effort.

Leaving aside rivalry in the product market, I first show that the optimal financial contract combines debt and equity. In good states financiers receive a fixed claim (debt) plus a constant fraction of remaining profits (outside–equity) the size of which is determined by the intensity of monitoring. The firm is punished only if it defaults on its debt and sanctions become more severe as the shortfall gets larger. The optimal capital structure trades off equity related cost of monitoring and debt related cost of sanctions in order to minimize the agency cost of external financing. Since most models of security design obtain pure debt as optimal financial contract this result may be of some interest in its own.

If the firm operates in duopoly it will worry about the rival's response to its capital structure. For the sake of the argument assume that in case of punishment funding is terminated, the firm has to suspend its operation and the rival gains monopoly. Since this happens with increasing probability as the firm's profits decrease, the rival has an incentive to prey, e.g. to deliberately set prices or quantities to cause harm. By punishing the firm for poor performance the optimal incentive contract also rewards the rival for being nasty. Now there is an additional trade off between solving the incentive problem at minimal cost and deterring predation. Not surprisingly, the firm will respond to the threat of predation by decreasing its leverage, thereby reducing the need for sanctions at the cost of intensified monitoring. This result confirms the widely held belief, that equity strengthens the firm in competition.

However, if the rival is prepared to forego immediate profits in order to increase the probability that the firm leaves the market, the optimal contract is not necessarily of the simple debt–equity type. It may be worthwhile to shut the firm down when performance is at its best in order to reward the rival for being nice. The increase in current profits may outweigh the expected loss of future profits. While such a feature appears rather odd in view of the empirical evidence on financial contracting it is a distinct theoretical possibility.

Our main result, that equity strengthens the firm in oligopoly, contrasts with those of Brander & Lewis (1986), and Showalter (1995) who consider debt as a tool of self–commitment. Leaving aside agency problems, this literature argues that debt is good in oligopoly, provided that an increase of capacity (or price) makes profits more risky. If the condition is met, the firm, by increasing its leverage, commits to follow a more risky strategy, implying an upward shift of its own reaction function. In my paper in the contrast, capital structure moves the *rival's* reaction function but it has no effect on the firms own behaviour.

That debt, by increasing the probability of bankruptcy and exit, invites predation is central to Poitevin's (1989) analysis of financing under asymmetric information. In his signaling model low cost firms attempt to convey their superiority to the capital market by taking up debt although this is followed by harmful predation. Whereas Poitevin (1989) takes financial contracts as exogeneously given, I derive them from the fundamentals of the problem. As a result the probability of exit is smaller than one when the firm defaults only by a small margin. Another difference is that contracting takes place under symmetric information.

The paper is closely related to Bolton & Scharfstein (1990) — in some sense it can be seen as an extension of their work. These authors conclude that: 'there is a tradeoff between deterring predation and mitigating incentive problems; reducing the sensitivity of the refinancing decision discourages predation, but exacerbates the incentive problem' ... '*The contract that minimizes agency problems, maximizes the rival's incentives to prey.*'[1] My model differs by introducing the possibility of equity–finance and by allowing cash–flow to be distributed on a continuum. With these changes I am able to generalize their first result and relate it explicitly to the firm's choice of capital structure. However, their second claim is rejected as an artifact of their assumption of a two–point distribution of cash–flow. In Bolton & Scharfstein (1990) the optimal contract responds to predation by either decreasing the probability of disruption in bad states or by increasing it in good states, depending on whether financiers gain or loose through termination. Here it is assumed that financiers are indifferent as to whether to terminate funding or not. Nevertheless both results are possible and may occur even simultaneously.

Section 2 presents the basic setup and shows that, without predation, the optimal financial contract can be obtained by a combination of debt and equity. In section 3 a rival in the product market is introduced. It is shown that (i) the 'naive' incentive contract invites predation, (ii) responding to the threat of predation, the firm will want to substitute debt for equity and (iii) if the simple debt–equity feature is lost, there is a positive probability of the firm leaving the market when performance is at its best. In section 4 the main results are illustrated with the help of a specific example. The proofs are in the appendix.

[1]Bolton & Scharfstein (1990), p. 93 and p. 101 (emphasis in the original).

2 The Agency Problem

Consider a risk–neutral firm with access to a profitable technology but without liquid funds. The project requires investment I and generates uncertain gross profits (before financing cost) $\pi \in \Pi \equiv [\underline{\pi}, \bar{\pi}]$ which is distributed with cumulative probability F and density f. In order to raise funds for the investment I the firm turns to a competitive capital market. Risk–neutral financiers acquire a 'share' $s(\pi)$ of the project's return. Since the firm has no liquid wealth, payments to financiers cannot exceed gross profits, $s(\pi) \leq \pi$. While investment fully depreciates the technological opportunity remains. If the firm is able to continue its operation after the first period it earns an income (net of the cost of renewed investment) with present value L. However, the technology is not transferable, hence L is lost if the firm is forced to abandon production. We follow Bolton & Scharfstein (1990) in assuming that old financiers can disrupt the operation of the firm conditional on performance, e.g. by seizure of assets or by imposing covenants on investment. For simplicity let $\beta \in [0, 1]$ denote the contracted probability that funding is terminated and the firm is closed down. Note, that disruption harms the firm without benefiting its financiers. We therefore, assume that the two sides are able to commit to ex–post inefficient outcomes.[2]

External funding is burdened with an agency problem because π is not contractible. After observing π the firm can choose to appropriate resources leaving only $\tilde{\pi} \leq \pi$ to be shared.[3] In one form or the other this is a standard assumption in the principal agent literature on financial contracting. If the firm is run by a manager, profits may be appropriated by diverting resources to managerial consumption in form of perquisites, pet projects, empire building etc. If the owner is in command, he may acquire a supplier and manipulate prices to fleece the company.

[2]Bolton & Scharfstein (1990) explicitly work out a two–period model to derive the losses (and possible gains) from disruption. In their particular set–up the incentive contract is renegotiation–proof, provided financiers prefer termination over continuation.

[3]The possibility of faking good results, $\tilde{\pi} > \pi$, is ignored in order to streamline the exposition. Since the firm has no incentive to do so at the optimal contract, the results would not change when faking is feasible.

Much of the related literature assumes that the firm is indifferent between appropriated resources: $(\pi - \tilde{\pi})$, and 'official income': $(\tilde{\pi} - s(\tilde{\pi}))$. Here we take into account that appropriation can be made difficult. Constraints imposed by the corporate charter or by organizational design, monitoring, accounting principles, restrictions on ownership stakes with business–partners etc. help to alleviate the incentive problem by increasing the cost of appropriation. For simplicity, we introduce appropriation cost which are proportional to the amount of resources appropriated: $\alpha \cdot (\pi - \tilde{\pi})$. Hence the firm's share of returns is given by: $\tilde{\pi} - s(\tilde{\pi}) + (1-\alpha)(\pi - \tilde{\pi})$. The loss–factor $\alpha \in [0,1]$ indicates how tight constraints are. $\alpha = 0$ indicates a very loose regime in which the firm's funds can be diverted into private consumption on a one to one basis. For $\alpha = 1$ on the contrary, the net–gain of appropriation is zero and the incentive problem disappears.

However, imposing constraints on the firm's control of resources has its own drawbacks which are captured by a cost of $m(\alpha)$. To simplify the exposition these shall be borne by the financiers. Obviously, m includes some cost of monitoring and accounting to prevent theft. It will therefore, be referred to as 'monitoring cost' which, however, should not be taken too literally. When a new project is separated from existing business to prevent cross–subsidization, the cost may consist of foregone economies of scale and scope. To obtain a well–behaved problem it is assumed that: $m'(0) = m(0) = 0$, $m' \geq 0, m'' > 0$, and $\lim_{\alpha \to 1} m(\alpha) = \infty$.

Since external financing becomes impossible when the firm never pays anything back, both sides will want to discourage appropriation. This can be achieved in two distinct ways. Financiers may monitor the firm's operations and curtail its control of resources to make appropriation more difficult. Alternatively, they may simply punish the firm by destroying its non–liquid wealth. The financial contract consists of a payment schedule $s(\pi)$, a disruption schedule $\beta(\pi)$, and a monitoring intensity α. The timing is as follows:

1. the contract determines α, β, s,
2. financiers deliver I and α is implemented,
3. the firm observes π and decides how much to leave $\tilde{\pi}$,

4. β, s are implemented,

The first–best contract (with π being contractible) would set $\alpha = 0$ and $\beta(\pi) = 0$, to avoid the cost of monitoring m respectively punishment βL, and use any s with $s(\pi) \leq \pi$ to fulfill the financiers' participation constraint. With the exception of $s = \underline{\pi}$, $\forall \pi$, such a contract will not be incentive compatible. To prevent the firm from cheating the contract has to obey the following incentive constraint:[4]

$$\pi - s(\pi) - \beta(\pi)L \geq \pi - s(\tilde{\pi}) - \alpha \cdot (\pi - \tilde{\pi}) - \beta(\tilde{\pi})L \qquad \forall\, \pi, \tilde{\pi} < \pi \tag{1}$$

It ensures that the firm's final wealth when sharing honestly, $\tilde{\pi} = \pi$ (the left side) is not smaller than when diverting $\pi - \tilde{\pi} > 0$ to its private ends (the right side).

When issuing financial claims the firm will solve the following program:
PROGRAM 1.

$$\max \int_{\underline{\pi}}^{\bar{\pi}} [\pi - s(\pi) - \beta(\pi)L] f \, d\pi \tag{P.1}$$

s.t.

$$\int_{\underline{\pi}}^{\bar{\pi}} s(\pi) f(\pi) \, d\pi \geq I + m(\alpha) \tag{PC.1}$$

$$s(\pi) \leq s(\tilde{\pi}) + \alpha(\pi - \tilde{\pi}) + (\beta(\tilde{\pi}) - \beta(\pi))L \quad \forall\, \pi, \tilde{\pi} < \pi \tag{IC.1}$$

$$s(\pi) \leq \pi \qquad \forall\, \pi \tag{WC.1}$$

$$\beta(\pi) \in [0, 1]; \tag{R.1}$$

Pareto–efficiency requires that the firm's expected profit is maximized subject to the constraint that financiers receive their reservation utility. The partici-

[4]Since cheating is costly, the requirement of zero–appropriation could be too restrictive. In principle, the optimal contract may allow for some appropriation in bad states in order to achieve higher payouts in good states. However, this is not possible with proportional appropriation cost, as can already be inferred from the linearity of (1).

pation constraint (PC.1) ensures that the investor's expected payoff covers his initial contribution, where the discount factor has been set equal to one to ease notation. Throughout the following it is assumed that a solution exists at which incentive constraint (IC.1) is binding and the firm's expected profits are positive.

For a start we take the control–regime α as exogenously given and show that the optimal choice of s and β is characterized by a parameter D, which can be interpreted as the face value of debt, while α yields the fraction of outside equity. Then, knowing that the optimal contract can be implemented with debt and equity, we can derive the optimal capital structure by optimizing over $\{\alpha, D\}$. The following proposition establishes the debt–equity–feature of the optimal financial contract.

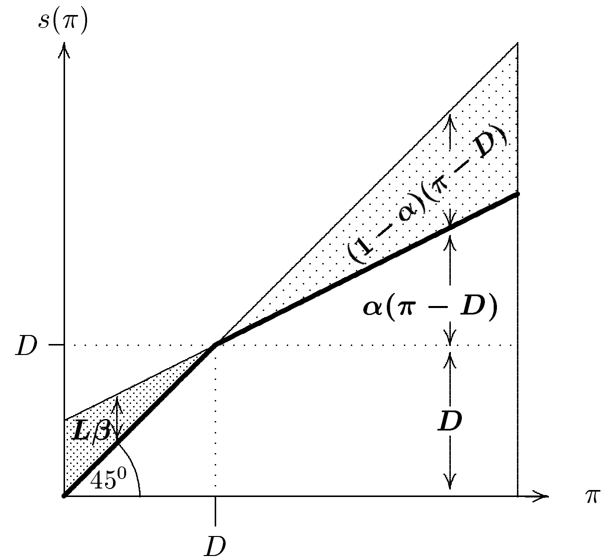
PROPOSITION 1 *For any given α, the optimal contract resembles a combination of debt with face value D and equity. As π falls below the threshold of D the probability that the firm is shut down increases and financiers obtain the whole output. For π larger than D the firms stays alive with certainty and financiers recover a fixed payment equal to D plus a share α of the residual $\pi - D$. Formally:*
If $\{s^o, \beta^o\}$ solves program 1, then $\exists\ D \geq 0$ such that

$$s^o(\pi) = \begin{cases} \pi; & \pi < D \\ D + \alpha(\pi - D); & \pi \geq D \end{cases}$$

$$\beta^o(\pi) = \begin{cases} (1-\alpha)(D-\pi)/L; & \pi < D \\ 0; & \pi \geq D \end{cases}$$

A typical proposition 1 contract is illustrated in figure 1. The thick line illustrates payments to investors s^o. In the case of a uniform distribution of π, the white area below s^o measures expected returns for investors. The lightly shaded area to the right marks funds retained by the firm and the darkly shaded area measures expected loss of income due to disruption. Three features of this contract invite the interpretation in terms of risky debt and equity: (i) operations are disrupted only if a fixed, hence debt–like, claim cannot be served.

Figure 1: Debt–Equity Feature of the Financial Contract



(ii) In case of 'insolvency' financiers recover as much as possible. (iii) In good states financiers do not interfere and obtain a fixed sum and a constant, hence equity–like, share α of the residual. The debt–equity contract minimizes the cost of disruption for any given α and expected income of the financiers.[5]

Obviously, proposition 1 keeps us close to the established literature on optimal financial contracts. With the interpretation of β borrowed from Bolton & Scharfstein (1990) we obtain Diamond's (1984) optimality of pure debt–financing as the limiting case for $\alpha \to 0$. For α exogenously determined, the optimal contract maintains a fixed relation between inside and outside equity.

[5]While s can be conceived as a combination of debt and equity, there is nothing in the model to require that s should in fact be divided into distinct claims, debt and equity, that payment in the control region accrues exclusively to debt–holders, or that residual control rights should be attached in any particular manner to these securities. This lack of explanatory power, however, should not be seen as a weakness of the framework — quite to the contrary. The underlying assumption that commitment to an ex–post inefficient allocations is possible can now be justified by arguing that these additional features can be used to prevent renegotiation and motivate a group of financiers to carry out the 'punishment'. Among recent papers explaining these additional features in this spirit are Berglöf & von Thadden (1994), Bolton & Scharfstein (1996) and Dewatripont & Tirole (1994).

Any increase in I would have to be met by increasing D. Hence, risky debt would be the marginal source of finance and the firm's leverage would be trivially determined by the amount of external finance needed. This view may be adequate when day–to–day financial decisions are concerned. However, for an investigation into the strategic aspects of financial contracting, it would be too narrow if deliberate changes in the capital structure, such as going public or undergoing a leveraged buyout, are ruled out. To determine the optimal capital structure we use proposition 1 to rewrite the contracting problem in terms of $\{\alpha, D\}$:

PROGRAM 2.

$$\max_{\alpha,D,\lambda} U(\alpha, D; F) + \lambda[V(\alpha, D; F) - I] \tag{P.2}$$

where the expected payoff of the firm U, and its financiers V are given as:

$$U(\alpha, D; F) = (1-\alpha)\int_{\underline{\pi}}^{\bar{\pi}} (\pi - D)\, dF$$

$$V(\alpha, D; F) = \int_{\underline{\pi}}^{D} \pi\, dF + \int_{D}^{\bar{\pi}} (D + \alpha(\pi - D))\, dF - m(\alpha)$$

and λ denotes the Langrange–multiplier associated with the participation constraint. Let $\{\alpha^o, D^o, \lambda^o\}$ solve the program. Provided that $1 > (1-\alpha^o)(D^o - \underline{\pi})/L$, which is assumed to be the case, it will also solve program 1. In this case (R.1) is not binding and the other constraints are fulfilled by construction. Then the implications for the optimal use of debt and equity can be summarized as follows:

PROPOSITION 2 *The firm issues debt and equity, $\alpha^o > 0$, $D^o > \underline{\pi}$. Agency cost drive a wedge between the cost of internal and external funds, i.e. the shadow price of external funds is larger than one, $\lambda^o = 1/(1 - F(D^o)) > 1$. The optimal capital structure balances the marginal gains and cost of monitoring according to:*

$$F(D)(1 - F(D)) \cdot (E[D - \pi | \pi < D] + E[\pi - D | \pi > D]) = m'(\alpha) \tag{2}$$

with E denoting the expectation–operator.

There are two kinds of benefits from monitoring in this model. First, it allows for a higher payment to financiers when the firm is solvent. This effect is related to the second conditional expectation in equation (2). In addition, the firm enjoys a lower probability of disruption in case of insolvency, which shows up in the first conditional expectation.

The model, described so far, provides a consistent and convenient framework for the analysis. Knowing that debt and equity are in fact optimal instruments, we can obtain an interior solution for the choice of capital structure. This result may be of some interest in its own, given that arguments put forward to explain the relevance of capital structure often favour border solutions.

3 Predation in the Product Market

So far the product market has been ignored. In an oligopoly however, the firm might worry about possible reactions of its rivals. To keep things simple, consider a single risk–neutral competitor. Let ρ denote the rival's strategic variable (price, advertising, capacity).[6] As a matter of definition an increase in ρ improves the probability distribution of the prey's profits $F(\pi;\rho)$ in the sense of first order stochastic dominance. Hence, high values of ρ indicate high prices or low capacities, i.e. a soft rival. To facilitate the exposition, it is assumed that $F_\rho(\underline{\pi},\cdot) = F_\rho(\bar{\pi},\cdot) = 0$ (invariant support) and $F_\rho < 0$, $\forall\, \pi \in (\underline{\pi},\bar{\pi})$.

In a simple one–period set–up, there are no gains from hurting the competitor as such (the same would be true with perfect capital markets). As a benchmark case, denote the corresponding profit of the rival $r(\rho)$ and define ρ^o by $\partial r/\partial\rho = 0$, that is the optimal action, if there are no strategic gains of predation. Then $\rho < \rho^o$ indicates predation i.e. prices (quantities) which are lower (higher) than optimal in a one–shot game.

Such a deviation from narrow–minded static optimization is warranted if the

[6] Predation may take many forms e.g lowering prices, increasing capacity, changing product quality, raising expenditure for advertising etc. Since many of these activities are difficult to measure exactly, it is ruled out that financial contracts explicitly depend on the rivals behaviour.

rival's future profits are related to the firms current profits. As in Bolton & Scharfstein (1990) and Poitevin (1989) we assume that the rival earns a (temporary) monopoly profit with discounted value $M \geq 0$ if the firm is forced to suspend operations. Since this happens with probability β the rival will choose ρ in order to maximize:

$$R(\rho) = r(\rho) + M \int_{\underline{\pi}}^{\bar{\pi}} \beta(\pi) f(\pi, \rho)\, d\pi$$

It is assumed that this problem is well behaved. Note, that other sanctions such as sacking the management, selling subsidiaries, forcing a reduction of capacity etc. may also weaken the firm in product market competition. While $R(\rho)$ would probably be non–linear in $E[\beta]$ our main results would still be true, provided that the rival's gain is larger the harsher the punishment is.

From the first order condition it can easily be established that the optimal incentive contract does give rise to predation.

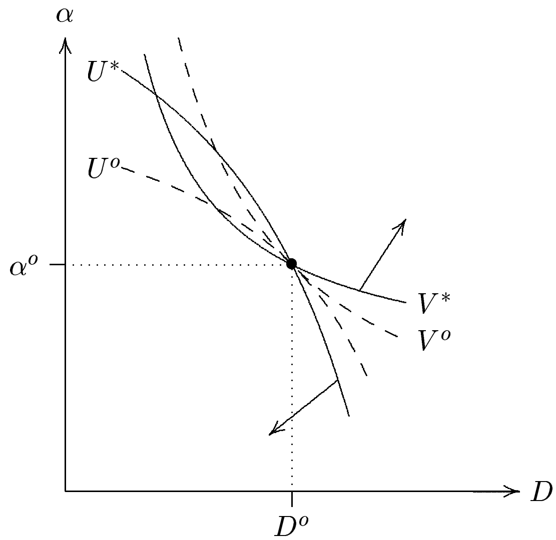
PROPOSITION 3 *The first best agency contract will induce predation if the rival was to gain through a disruption in the firm's financing:*
$M > 0 \implies \rho^*(\beta^o, s^o) < \rho^o$.

The result is not surprising so far as the incentive contract by punishing the firm for poor performance also rewards the rival for predatory behaviour. How should the firm respond to the threat of predation when designing its financial contract? Intuitively, it ought to reduce the probability of disruption in bad states. By decreasing the sensitivity of sanctions to performance it weakens the rival's benefits from poor performance, hence his incentives to prey. We address the issue in two steps. First, it is assumed that the firm will stick to the debt–equity feature of external financing. It may, however, adjust its leverage. Second, we ask, whether and how the firm might want to deviate from this familiar feature of the optimal contract.

To derive the optimal capital structure we turn again to program 2. This time, the solution, denoted $\{\alpha^*, D^*\}$, takes into account that F depends on ρ^* which is implicitly defined by the first order condition:

$$0 = r'(\rho^*) + (1 - \alpha)\frac{M}{L} \int_{\underline{\pi}}^{D} F_\rho(\pi, \rho^*)\, d\pi \qquad (3)$$

Figure 2: Leverage with Predation



Since $d\rho^*/dM < 0$ we may use M as a parameter to measure the intensity of the rival's incentives to predate. For $M = 0$ we obtain the non–predation case: $\rho^* = \rho^o$ and $\{\alpha^*, D^*\} = \{\alpha^o, D^o\}$. For a small increase of M the usual comparative statics in M, evaluated at $M = 0$ yield:

PROPOSITION 4 *If the threat of predation is small and if the optimal contract maintains the debt–equity feature then the firm will decrease its leverage.*
For M sufficiently small $\Longrightarrow \alpha^* > \alpha^o$ and $D^* < D^o$.

The reason for the decrease in leverage can be inferred from the indifference curves of the firm and its financiers in the α–D–space. With the incentive contract without predation, $\{\alpha^o, D^o\}$, these are tangent to one another and decreasing. They are plotted as dashed curves in figure 2. Predation steepens the firm's indifference curves ($U^o \to U^*$) while flattening those of its financiers ($V^o \to V^*$). An increase of D leads to more aggressive competition, $\rho^*_D < 0$, hence it is more costly for the firm and less valuable for its financiers. In order to maintain indifference, the firm requires a larger reduction of α while its financiers can only grant a smaller reduction. This wedge between the rates of substitutions is further amplified because a decrease of α also encourages the rival to prey, $\rho^*_\alpha > 0$. It is therefore, more costly for the financiers and less worthwhile for the firm as compared to a situation without predation. Like a tax on debt and a subsidy for equity predation tilts the trade–off between equity related cost of monitoring and debt related cost of disruption against debt–financing. With the solidly plotted indifference curves, both sides favour a shift towards the 'lens' of pareto improving contracts in the north–west, hence towards more equity–based financing.

While the substitution effect clearly points to less debt–financing there is an additional 'income' effect which, in general, renders the overall impact of predation ambiguous. By turning nasty the rival decreases expected profits. The expected utilities associated with U^* and V^* are lower than those of U^o and V^o. Without a change of contract, the participation constraint of the financiers would be violated. Hence, the 'income' effect requires an increase of α and D. The latter might offset the substitution effect.

Proposition 4 supports the common intuition, that a firm should decrease its leverage when threatened with predation in the product market. Decreasing D while increasing α makes exit less likely and less sensitive to performance which in turn makes predation less rewarding. However, the result is based on the assumption that the debt–equity feature of the financial contract is maintained in the presence of a competing rival. In the rest of this section we ask how the optimal contract might look like if this assumption is not warranted.

As in program 1 we are interested in the optimal disruption schedule $\beta(\pi)$ and the corresponding payment $s(\pi)$ taking α as exogeneously given. The non–strategic debt–equity contract minimized expected disruption cost given α, I and F by setting $\beta^o(\pi)$ as small as compatible with the incentive constraint. Now we account for the dependence of $F(\pi, \rho^*)$ on β by the way of the rival's choice of ρ^* which is implicitely defined by the first order condition $R' = 0$. The rival uses ρ to shift probability mass towards those levels of profit for which the probability of disruption is high. This provides an additional motive to make β small for the lowest profit. However, as profits increase the reason to minimize β becomes weaker because the rival's response is getting less harmful. By raising β when profits are high the firm actually rewards the rival for acting to its advantage. Whether softening the rival in the product market is worthwhile will depend on the cost of disruption L and the details of the transmission from β into F.

In order to obtain further insights into this complicated trade–off we have to assume that the family of distributions $F(\cdot;\rho)$ obeys the monotone likelihood ration property (MLRP):

$$\frac{d}{d\pi}\left(\frac{F_{\rho\pi}}{F_{\pi}}\right) > 0 \tag{4}$$

To gain an intuitive understanding of the condition, assume that ρ is unobservable. If (4) holds, then for any a–priori believes about ρ, the observance of $\pi_2 > \pi_1$ would make it more likely that π_2 has been drawn from a more favourable distribution, hence a distribution characterized by a higher ρ.[7]

MLRP has rather strong and somewhat surprising implications for the optimal way to discourage predation. One might expect the firm to reduce the sensitivity of disruption with respect to performance by decreasing the slope of β. For any given α, this would require that β is increased over β^o for some intermediate levels of profit. However, MLRP implies that predation is discouraged more effectively, the higher the firm's profits are, at which an increase of β takes place. If the firm is prepared to incur any additional disruption cost at all, it should do so by raising the probability of disruption when profit is at its highest.

To develop the argument in more detail, let's start, by ignoring the incentive constraint of program 1. Then, the best way to deter predation is to shut down the firm with probability one in some upper range of profits.

PROPOSITION 5 *If the MLRP is fulfilled, the contract which minimizes (maximizes) predation for any given expected disruption cost $E[\beta]L$ sets $\beta = 1$ for all π above (below) a threshold value $\hat{\pi}$ and zero otherwise.*

If the firm can commit to exit the market in good states it faces yet another trade off. On the one hand it may ensure a nicer treatment from his rival who

[7]See Milgrom (1981) for a comprehensive discussion of the concept.

will be the more responsive the larger his gains M from obtaining a temporary monopoly are. On the other hand it foregoes L with probability $1 - F(\hat{\pi}, \rho^*)$. Whether rewarding the rival for being nice can be made worthwhile through a suitable choice of $\hat{\pi}$ depends on the details of the strategic interaction and the parameters of the problem, in particular the size of M and L.[8]

The trade-off is somewhat more delicate when financial contracting is burdened with the agency problem. As can be inferred from the incentive constraint (1) the upward jump of β at $\hat{\pi}$ has to be matched by a downward jump of s with size L in order to prevent the firm from cheating. Nevertheless, the optimal contract is obtained by blending the debt–equity feature of the non–strategic incentive contract (proposition 1) with the exit–at–the–top feature of the predation minimizer (proposition 5).

PROPOSITION 6 *If strategic rivalry justifies a deviation from the simple debt–equity feature then the firm will be closed down with certainty when profit is at its highest. Formally:*
Let $\{\hat{\beta}, \hat{s}\}$ solve program 1 for a given α taking into account the dependence of F on the disruption schedule $\beta(\pi)$ by the way of the rivals choice of ρ. If the MLRP is fulfilled, then $\exists$ $\{D, \hat{\pi}\}$ such that:

$$\hat{s}(\pi) = \begin{cases} \pi; & \pi < D \\ D + \alpha(\pi - D); & D \le \pi < \hat{\pi} \\ D + \alpha(\pi - D) - L; & \hat{\pi} \le \pi \end{cases}$$

$$\hat{\beta}(\pi) = \begin{cases} (1-\alpha)(D-\pi)/L; & \pi < D \\ 0; & D \le \pi < \hat{\pi} \\ 1; & \hat{\pi} \le \pi \end{cases}$$

[8]Comparing the incentive contract of proposition 1 and the predation–maximizer of proposition 5 shows that, while inviting predation, the optimal agency contract does not maximize the incentives to predate. This suggests that Bolton & Scharfstein's (1990) second claim mentioned in the introduction does not generalize beyond their two–point distribution of profits. Without MLRP very little can be said about the predation–maximizing contract. Generically however, it will be different from the optimal incentive contract.

With this partial result the efficient financial contract can be characterized by three parameters $\{\alpha, D, \hat{\pi}\}$. Generally, border solution for these parameters may be optimal. In particular, it may be optimal to set $\hat{\pi} > \bar{\pi}$, which would be equivalent to a pure debt–equity contract. If the contract, however, deviates from the debt–equity feature, it will do so in a rather strange way, that is by interrupting the firm's operations when performance is at its best. Since we do not observe such contracts, it appears as if the cost of rewarding rivals in this manner usually outweigh its benefits.[9] Theoretically however, it is easy to construct examples in which softening a rival does pay off as the next section will show.

4 Example

So far the strategic interaction in the product market has been analyzed in very abstract terms. To illustrate the qualitative results of the previous sections we now consider a specific example of product market rivalry which is based on the model of Cournot competition applied by Poitevin (1989). Unfortunately, when analyzed within an optimal contracting framework even very simple models defy an explicit analytical solution, which is why I have to resort to numerical calculations.

At the first stage the firm raises external funds to finance a fixed set–up investment. After financial arrangements are made the firm and a rival produce quantities x, respectively y, at zero marginal cost to serve an inverse market demand $p(x + y)$. Thereafter operating returns realize which are subject to an additive random shock ϵ with support $[-e, +e]$. A possible interpretation is that fixed cost are uncertain. At the final stage profit, given as $\pi = p(x + y)x + \epsilon$, is shared between the firm and its financiers according to the financial contract. Recall that the firm's penalty is L and the rival's profit is $R(y) = p(x + y)y + M\mathrm{E}[\beta]$, with M denoting his gain if the firm is punished. For algebraic convenience it is assumed that demand is linear and the distribution of ϵ is uniform with zero mean, hence $\mathrm{E}[\pi] = \pi(x, y)$. With

[9] One may also find it more difficult to commit oneself to terminate funding when profits are high.

demand normalized to $p(x+y) = 9(1-(x+y))$ it is easy to verify that the normal Nash–Cournot solution entails: $x^c = y^c = 1/3$ and $\pi(x^c, y^c) = 1$.

First we turn to the pure debt–equity contract. For any given values of α and D, the probability of closing down the firm due to insolvency is

$$\mathrm{E}[\beta] = \frac{(1-\alpha)}{L} \cdot \frac{(D+e-\pi(x,y))^2}{4e}$$

From first order conditions for the optimal choice of x and y one can derive a cubic–equation for the product market equilibrium. Its solution, denoted $\{x^*, y^*\}$, is determined by three parameters e, M and L, and two financial variables α, D. At the financing stage the firm sets $\{\alpha, D\}$ anticipating the product market equilibrium in order to maximize its expected share of profit subject to the capital market constraint that expected payout has to cover investment cost I. With a monitoring technology $m(\alpha) = \kappa\alpha/(1-\alpha)$, we would have two more parameters κ and I. Given the complexity of the expressions for $\{x^*, y^*\}$, first order conditions for α and D cannot be solved for an explicit solution relating the optimal capital structure $\{\alpha^*, D^*\}$ to the parameters $\{e, L, M, I, \kappa\}$. However, with their help it is easy to obtain numerical solutions, once the parameters are specified and the following example may be instructive:

$$e = 0.9, \quad L = 1, \quad M = \text{variable}, \quad I = 0.7, \quad \kappa = 0.05$$

The results of the computations are presented in table 1. The entries in the first column stand for a firm that does not have to worry about predation because $M = 0$. They correspond to the solution $\{\alpha^o, D^o\}$ as characterized in proposition 2. The firm issues debt with value 0.519 ($D = 0.584$) and equity with value 0.225 ($\alpha = 0.468$). In the product market we have the symmetric Cournot solution which results in a probability of 0.035 that the firm has to leave the market due to default. The associated loss in wealth and monitoring cost of 0.044 drive the shadow cost of external funding up to 1.37.

Now we turn to the case of predation and set $M = 0.3$. If the firm would not adjust its financial contract the rival would increase his quantity from 0.333 to 0.343 and the firm would produce only 0.328 in product market equilibrium (second column). Its profits would decline to 0.97 and the probability of exit

Table 1: Numerical Example

	optimal capital structure (no predation)	naive contract with predation	optimal capital structure (predation)	reward only	reward with incentive constraint
M	0	0.3	0.3	0.3	0.3
α	0.468	0.468	0.57		0.522
D	0.584	0.584	0.547		0.442
$\hat{\pi}$				1.98	1.96
λ	1.37		1.58		1.35
x	0.333	0.328	0.33	0.353	0.351
y	0.333	0.343	0.341	0.294	0.299
π	1.00	0.97	0.978	1.121	1.106
exit / default	0.035	0.039	0.026		0.007
exit / reward				0.026	0.027
U	0.221	0.206	0.185	1.109	0.318
V	0.7	0.682	0.7		0.7
value equity	0.225	0.215	0.28		0.354
value debt	0.519	0.51	0.486		0.427
$m(\alpha)$	0.044	0.044	0.066		0.054

would increase from 0.035 to 0.039. However, this financial contract is not feasible. Due to the income effect of predation it violates the capital market constraint (financiers expected returns drop to $0.682 < I + m(\alpha) = 0.744$). As suggested in proposition 4 the firm would deleverage in response to predation (provided the debt–equity feature is maintained). The corresponding figures are given in the third column. By increasing α and decreasing D the firm strengthens its position in the product market and improves profits slightly from 0.97 to 0.978. Compared to the no–predation case however, the firm's expected payoff suffers badly from decreased profitability and increased agency cost. It declines from 0.221 to 0.185.

The example also illustrates the possibility of rewarding the rival for friendly behaviour. Consider first the predation minimizer characterized in proposition 5 which terminates funding when profits pass a treshhold $\hat{\pi}$ (fourth column in the table). It can be shown that rewarding the rival in this manner pays off, provided that $L < 16/3e - 4/9M$, which is clearly the case in our example.[10] For the optimal $\hat{\pi} = 1.98$ market shares shift so much in favour of the firm that expected profits increase by 0.121 over the Cournot outcome — which is enough to compensate for 0.026 in expected loss of wealth. Finally, column five gives the results of the debt–equity contract when blended with a predation–minimizing contract. Compared to the pure debt–equity–contract the firm's expected payoff U increases from 0.221 to 0.318 which is again higher than if the rival would not respond to the probability of exit.

5 Concluding Remarks

Most formal models in finance seem to favour debt (if anything at all), whether for tax reasons, the provision of high powered incentives, as a signaling device or as a tool of strategic commitment in the product market. The preceding

[10] With a uniform distribution, the rival will choose either the Cournot–quantity or play soft, depending on x and $\hat{\pi}$. In both cases his optimal quantity does not depend on $\hat{\pi}$. Since his reaction function is not continuous, an equilibrium in the market–game does not exits for all values of $\hat{\pi}$. The firm is assumed to set $\hat{\pi}$ as high as possible (in order to keep $(1-F(\hat{\pi}))L$ low), while ensuring an equilibrium in which the rival is soft.

analysis runs into the opposite direction. It shows that, a highly leveraged capital structure makes the firm vulnerable to predation in the product market. When external funding is burdened with agency problems, debt requires some sort of 'punishment' in case of default in order to be incentive compatible. If a rival benefits from the firm being punished for poor performance, i.e. if the firm is closed down when defaulting on debt, he will compete more aggressively to increase the probability of this to happen. Equity requires monitoring and constraints on the firm's control of resources. Even if the rival gains from equity related cost, such as a decrease in flexibility, he has no incentive to prey, since these cost are largely independent of the realization of profits. Thus, a strong equity base offers protection against hostile competition. By combining agency problems in financial contracting with product market rivalry, the model does not only provide the missing link in the 'long–purse' story, it can also explain why low powered incentive schemes are so common.

However, when the issue is analyzed as a problem of security design, the whole story takes an unexpected turn. The debt–equity feature is optimal in a simple firm–financier–agency setting. Adding a rival who responds to the termination of funding raises new possibilities. The optimal contract may force the firm out of business when its performance is at its best. This encourages a rival to reduce quantities or to increase price in order make higher profits more likely. When rivals may turn nasty, they can also be changed to nice.

6 Appendix

Proof of Proposition 1

Suppose there exist a contract $\{\tilde{s}, \tilde{\beta}\}$ for which $\tilde{s}$ is different from s^o on a subset $\tilde{\Pi}$ with $\int_{\tilde{\Pi}} dF \neq 0$. Since the wealth constraint binds for $\pi \leq D$, there exist an $\pi_0 \in [D, \bar{\pi}]$ for which $\tilde{s}(\pi_0) > s^o(\pi_0)$, otherwise the participation constraint would be violated. To obey the incentive constraint, however, the probability of termination has to be raised by at least $(\tilde{s}(\pi_0) - s^o(\pi_0))/L$ for $\pi \in [\underline{\pi}, D]$. While $\tilde{\beta} > \beta^o$ on $[\underline{\pi}, D]$ it cannot be lower on $[D, \bar{\pi}]$. Hence, the expected cost of disruption is strictly higher with $\{\tilde{s}, \tilde{\beta}\}$. Since the expected payments to financiers cannot be smaller, the contract is inferior. $\square$

Proof of Proposition 2

All claims follow easily from first order conditions.

Proof of Proposition 3

By proposition 1 the rivals profit can be rewritten as:

$$r(\rho) + M(1-\alpha) \int_{\underline{\pi}}^{D} [(D-\pi)/L] f(\pi, \rho) \, d\pi$$

Recall that the support is fixed, hence $F_\rho(\underline{\pi}, \rho) = F_\rho(\bar{\pi}, \rho) = 0$. By second order conditions $\rho^* < \rho^o$ requires

$$0 > M(1-\alpha) \int_{\underline{\pi}}^{D} [(D-\pi)/L] f_\rho(\pi, \rho) \, d\pi$$

Partial integration yields:

$$0 > (1-\alpha)(M/L) \int_{\underline{\pi}}^{D} F_\rho \, d\pi$$

which holds true because $F_\rho < 0$. $\square$

Proof of Proposition 4

Second order conditions for ρ^* being optimal require $\textsc{soc} \equiv r'' + (1-\alpha)M \int_{\underline{\pi}}^{D} F_{\rho\rho} d\pi < 0$. First order condition (3) defines the rival's responses to an increase of equity, respectively debt, as a pair of implicite functions:

$$\begin{aligned} \rho^*_\alpha(M) &= M \int_{\underline{\pi}}^{D} F_\rho d\pi / \textsc{soc} < 0, \quad \text{for } M > 0 \\ \rho^*_D(M) &= -(1-\alpha) M F_\rho(D) / \textsc{soc} > 0, \quad \text{for } M > 0 \end{aligned}$$

with:

$$\rho^*_\alpha(0) = \rho^*_D(0) = 0; \quad \frac{d\rho^*_\alpha}{dM}(0) > 0; \quad \frac{d\rho^*_D}{dM}(0) < 0 \tag{5}$$

Let $\mathcal{L}$ denote the Lagrange–function in program 2 and define:

$$\Psi \equiv -(1-\alpha) \int_{\underline{\pi}}^{\bar{\pi}} F_\rho d\pi + \lambda \left(- \int_{\underline{\pi}}^{D} F_\rho d\pi - \alpha \int_{D}^{\bar{\pi}} F_\rho d\pi \right) > 0$$

Then, first order conditions can be written as

$$\begin{aligned} \mathcal{L}_\alpha &= U^o_\alpha + \lambda V^o_\alpha + \Psi \rho^*_\alpha(M) = 0 \\ \mathcal{L}_D &= U^o_D + \lambda V^o_D + \Psi \rho^*_D(M) = 0 \\ \mathcal{L}_\lambda &= V^o - (I - W) = 0 \end{aligned}$$

where the superscript ^{o} indicates, that partial derivates do not take into account the impact on F. If evaluated at $M = 0$ we obtain the same conditions as without predation. Note that $V^o_D = (1-\alpha^o)(1 - F(D^o)) > 0$ and $U^o_\alpha = -U^o < 0$. From $\mathcal{L}_\alpha = 0$ follows $V^o_\alpha > 0$. In addition:

$$\mathcal{L}_{\alpha M} = \Psi \frac{d\rho^*_\alpha}{dM} > 0, \quad \mathcal{L}_{DM} = \Psi \frac{d\rho^*_D}{dM} < 0, \quad \mathcal{L}_{\lambda M} = 0$$

Going through the usual comparative statics and evaluating the expressions at $M = 0$ one obtains:

$$\begin{aligned} \text{sign } \left. \frac{d\alpha^*}{dM} \right|_{M=0} &= \text{sign } (\mathcal{L}_{\alpha M} (V^o_D)^2 - V^o_D V^o_\alpha \mathcal{L}_{DM}) > 0 \\ \text{sign } \left. \frac{dD^*}{dM} \right|_{M=0} &= \text{sign } (\mathcal{L}_{DM} (V^o_\alpha)^2 - V^o_D V^o_\alpha \mathcal{L}_{\alpha M}) < 0 \end{aligned}$$

Hence, for a small increase in M we may conclude that $\alpha^* > \alpha^o$ and $D^* < D^o$

□

Preparing Proposition 5 and Proposition 6

LEMMA 1 *With the monotone likelihood ratio property any shift of β which decreases β for some low profits and increases β for some higher profits, keeping the expected value of β constant, decreases predation:*
Let $\beta_\Delta = \beta_2 - \beta_1$ such that there $\exists\, \hat{\pi}$ with $\beta_\Delta \leq 0, \forall \pi < \hat{\pi}$ (with strict inequality on a subset with positive probability) and $\beta_\Delta \geq 0, \forall \pi \geq \hat{\pi}$ and $\int_{\underline{\pi}}^{\bar{\pi}} \beta_\Delta F_\pi \, d\pi = 0$, then (4) $\Longrightarrow \rho^*(\beta_2) > \rho^*(\beta_1)$.

Proof:
By second order condition ρ^* will increase if $dE[\beta_\Delta]/d\rho > 0$.

$$\begin{aligned}
\int_{\underline{\pi}}^{\bar{\pi}} \beta_\Delta F_{\pi\rho} \, d\pi &= \int_{\underline{\pi}}^{\hat{\pi}} \beta_\Delta F_{\pi\rho} \, d\pi + \int_{\hat{\pi}}^{\bar{\pi}} \beta_\Delta F_{\pi\rho} \, d\pi \\
&= \int_{\underline{\pi}}^{\hat{\pi}} \beta_\Delta F_\pi \frac{F_{\pi\rho}}{F_\pi} \, d\pi + \int_{\hat{\pi}}^{\bar{\pi}} \beta_\Delta F_\pi \frac{F_{\pi\rho}}{F_\pi} \, d\pi
\end{aligned}$$

Using $\int_{\underline{\pi}}^{\hat{\pi}} \beta_\Delta F_\pi \, d\pi = -\int_{\hat{\pi}}^{\bar{\pi}} \beta_\Delta F_\pi \, d\pi$ and denoting the integration index on the lower and higher interval l respectively h we obtain:

$$\begin{aligned}
\int_{\underline{\pi}}^{\bar{\pi}} \beta_\Delta F_{\pi\rho} \, d\pi &= \int_{\underline{\pi}}^{\hat{\pi}} \frac{\beta_\Delta(l) F_\pi(l)}{\int_{\underline{\pi}}^{\hat{\pi}} \beta_\Delta F_\pi \, d\pi} \cdot \left(-\int_{\hat{\pi}}^{\bar{\pi}} \beta_\Delta(h) F_\pi(h) \, dh \right) \cdot \frac{F_{\pi\rho}(l)}{F_\pi(l)} \, dl \\
&\quad + \int_{\hat{\pi}}^{\bar{\pi}} \beta_\Delta(h) F_\pi(h) \left(\int_{\underline{\pi}}^{\hat{\pi}} \frac{\beta_\Delta(l) F_\pi(l)}{\int_{\underline{\pi}}^{\hat{\pi}} \beta_\Delta F_\pi \, d\pi} \, dl \right) \frac{F_{\pi\rho}(h)}{F_\pi(h)} \, dh \\
&= \int_{\underline{\pi}}^{\hat{\pi}} \int_{\hat{\pi}}^{\bar{\pi}} \frac{\beta_\Delta(l) F_\pi(l) \cdot \beta_\Delta(h) F_\pi(h)}{\int_{\underline{\pi}}^{\hat{\pi}} \beta_\Delta F_\pi \, d\pi} \cdot \left[\frac{F_{\pi\rho}(h)}{F_\pi(h)} - \frac{F_{\pi\rho}(l)}{F_\pi(l)} \right] dl \, dh \\
&> 0
\end{aligned}$$

The expression is strictly positive, since the first term in the integral is strictly positive on a set with positive measure (and zero otherwise) due to the assumptions on β_Δ and the second term in brackets is strictly positive by MLRP. □

Proof of Proposition 5 (sketch)

Assume that the predation–minimizer would be different and use lemma 1 to show that predation can be further discouraged with no additional cost. □

Proof of Proposition 6

Assume that the optimal contract would be different from the proposition 6 contract on a non–degenerated subset of $[\underline{\pi}, \bar{\pi}]$. Denote the contract $\{\tilde{s}, \tilde{\beta}\}$, the associated action of the rival $\tilde{\rho}$, the expected payoff of the firm $\tilde{U}$ and of its financiers $\tilde{V}$. By constructing a superior contract we refute the alleged optimality of $\{\tilde{s}, \tilde{\beta}\}$.

First, consider the following transformation. For an arbitrary $\hat{\pi}$ replace the contract $\{\tilde{s}, \tilde{\beta}\}$ by a new contract $\{\hat{s}, \hat{\beta}; \hat{\pi}\}$ with $\hat{\beta} = \tilde{\beta} - \beta_\Delta$, $\hat{s} = \tilde{s} + s_\Delta$ and

$$\begin{aligned} s_\Delta(\pi) &= \beta_\Delta(\pi) L \\ \beta_\Delta(\pi) &= \begin{cases} \text{argmax } \tilde{s} + \beta_\Delta(\pi) L, & \\ \text{ s.t. } \hat{s} \leq \pi, \ \hat{\beta} \geq 0, & \text{for } \pi < \hat{\pi} \\ -1 + \tilde{\beta}, & \text{for } \pi \geq \hat{\pi} \end{cases} \end{aligned}$$

Below $\hat{\pi}$, the transformation makes β as small as possible while increasing s as much as needed to keep the firm indifferent. Above $\hat{\pi}$, β is set equal to 1, again with s being adjusted according to the incentive constraint. For any $\hat{\pi}$ the contract $\{\hat{s}, \hat{\beta}; \hat{\pi}\}$ already has the desired shape. For convenience denote the expected utilities with this contract conditional on ρ by $\hat{U}(\hat{\pi}, \rho)$, respectively $\hat{V}(\hat{\pi}, \rho)$. The contract fullfills the incentive and wealth constraints by construction. For a fixed $\tilde{\rho}$ it leaves the firm indifferent to the original contract: $\hat{U}(\hat{\pi}, \tilde{\rho}) = \tilde{U}$, $\forall \hat{\pi}$.

Now choose a particular $\hat{\pi}_0$ so that $E[\beta_\Delta | \tilde{\rho}] = 0$. Such a $\hat{\pi}_0$ exists because $E[\beta_\Delta | \tilde{\rho}]$ is continuously decreasing in $\hat{\pi}$, positive for $\hat{\pi} = \underline{\pi}$ and negative for $\hat{\pi} > \bar{\pi}$ (which would be equivalent to a pure debt–equity contract). Since the changes in β and s are proportional, this ensures that $E[s_\Delta | \tilde{\rho}] = 0$ which implies $\hat{V}(\hat{\pi}_0, \tilde{\rho}) = \tilde{V}$. Hence the contract $\{\hat{s}, \hat{\beta}; \hat{\pi}_0\}$ would also fullfill the participation constraint of the financiers, provided the rival would still select

$\tilde{\rho}$. However lemma 1 implies that ρ increases in response to the transformation, $\hat{\rho}(\hat{\pi}_0) > \tilde{\rho}$.

Since the firm's payoff is strictly increasing in π for $\pi > D$ and non–decreasing otherwise, it would strictly be better off with the new contract $\hat{U}(\hat{\pi}_0, \hat{\rho}(\hat{\pi}_0)) > \hat{U}(\hat{\pi}_0, \tilde{\rho}) = \tilde{U}$. However, for $\hat{\pi} < \bar{\pi}$ payment to the financiers is not monotone in π. Hence we have to consider two possibilies.

1. If $\hat{V}(\hat{\pi}_0, \hat{\rho}(\hat{\pi}_0)) \geq \hat{V}(\hat{\pi}_0, \tilde{\rho}) = \tilde{V}$ then $\{\hat{s}, \hat{\beta}; \hat{\pi}_0\}$ is already superior.

2. If $\hat{V}(\hat{\pi}_0, \hat{\rho}(\hat{\pi}_0)) < \hat{V}(\hat{\pi}_0, \tilde{\rho})$ we have to modify the contract further.

   With the pure debt–equity contract, $\hat{\pi} = \bar{\pi}$, payout is monotone in π, hence $\hat{V}(\bar{\pi}, \rho)$ is increasing in ρ. For all $\rho_1 > \tilde{\rho}$ it follows that: $\hat{V}(\hat{\pi}_0, \tilde{\rho}) < \hat{V}(\bar{\pi}, \tilde{\rho}) < \hat{V}(\bar{\pi}, \rho_1)$. The first inequality is due to an increase in expected payouts by $(1 - F(\hat{\pi}_0, \tilde{\rho}))L$. The second inequality results from the first order shift of F delivered by $\rho_1 > \tilde{\rho}$. Since $\hat{\rho}(\hat{\pi})$ is continuous in $\hat{\pi}$ and $\hat{V}(\hat{\pi}, \rho)$ is continuous in $\hat{\pi}$ and ρ, $\hat{\pi}$ can be increased over $\hat{\pi}_0$ to reach $\hat{\pi}_1 < \bar{\pi}$ for which either $\hat{\rho}(\hat{\pi}_1) = \tilde{\rho}$ or $\hat{V}(\hat{\pi}_1, \hat{\rho}(\hat{\pi}_1)) = \hat{V}(\hat{\pi}_0, \tilde{\rho})$ but $\hat{\rho}(\hat{\pi}_1) > \tilde{\rho}$. In the former case, financiers are better off by $(F(\hat{\pi}_1, \tilde{\rho}) - F(\hat{\pi}_0, \tilde{\rho}))L$ and the firm is indifferent. In the latter case, the financiers are indifferent but the firm is strictly better off.

Since either $\{\hat{s}, \hat{\beta}, \hat{\pi}_0\}$ or $\{\hat{s}, \hat{\beta}, \hat{\pi}_1\}$ is Pareto–superior, $\{\tilde{s}, \tilde{\beta}\}$ cannot be optimal. □

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