Free–Cash–Flow, Equity Finance and Corporate Control

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September 1997

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Abstract

This paper analysis optimal financial contracts which simultaneously provide pay–incentives and impose constraints on managerial control. When the manager is risk neutral the optimal contract is shown to resemble a combination of debt–like and equity–like securities. When circumstances are bad, investors interfere with managerial decision making in order to compensate for weak pay–incentives and managers may appropriate free–cash–flow. Since debtholders resume residual control in bad states, the model explains debtholders activism and shareholders passivity. Distorted incentives imply that performance will be particularly poor when firm potential is already low. We also consider the case of risk aversion and derive conditions for an under–investment bias.

Keywords: Free Cash Flow, Corporate Finance, Security Design
JEL class. G32, D82

1 Introduction

Motivation. The separation of ownership and control depends on financial contracts which stipulate payment and assign control rights to managers and financiers. Given that optimal managerial decision–making cannot be simply written into a contract, a first best solution would require the manager to be the residual claimant. It is well understood that such strong incentives have to be compromised on if she is wealth constrained or risk–averse or both. Another implication, however, has received less attention. If first best incentives cannot be achieved, delegation of control may also become distorted. If she who should be in charge cannot be made residual claimant, he who can be made residual claimant might retain some control — even if he lacks competence or time or both. By carefully curtailing managerial control financiers can make deviation from efficient behaviour less attractive. Hence, investors' activism may substitute for pay–incentives.[1]

This paper follows the free–cash–flow hypothesis in assuming that potential firm value is not–contractible. Hence, the manager may appropriate value through 'empire–building', consumption on the job, tolerating organizational slack etc. However, by constraining the management through accounting rules, separation of business and private assets, monitoring etc. financiers prevent simple theft and obtain contractible information about realized firm value. Appropriation, therefore, involves a cost over straight pay which can be exploited by incentive schemes. Contingent on performance financiers may also actively intervene to curb managerial control and discourage appropriation. We analyze optimal financial contracts which simultaneously provide pay–incentives and impose constraints on managerial control for various assumptions regarding the cost of continuous monitoring and state contingent interference, appropriation cost and risk preferences.

When the manager is risk neutral the optimal contract is shown to resemble a combination of debt–like and equity–like securities. It features high powered

[1] We couch the analysis in terms of a (female) manager and a (male) financier, also referred to as investor. However, with active investors, the distinction between both sides is less sharp than this terminology suggests.

incentives combined with unrestrained managerial control when performance is good. As performance worsens, pay–incentives lose power and interference by financiers gets strengthen. Hence, we provide an explanation for shareholder passivity and debtholder activity. With respect to pay related incentives, we motivate the use of option–plans in managerial compensation schemes and explain why firm performance is particularly poor when firm potential is already low. Since managerial consumption is convex under fairly general conditions, optimal contracts induce a preference for risk–taking.

We also characterize optimal contracts for a risk–averse manager. These, however, are difficult to relate to empirical features of financial securities. Finally, we investigate the impact of optimal finance on investment. Under mild regularity conditions, we confirm the traditional thesis, that agency cost, by driving a wedge between the cost of internal funds and uncollaterized external funds, result in underinvestment.

Literature. This paper combines elements of the literature on capital structure in the tradition of Jensen & Meckling (1976) with features of the literature on security design as initiated by Townsend (1979) and Gale & Hellwig (1986). According to the first approach, verifiable returns of an investment project are shared in order to provide a manager, acting under uncertainty, with incentives to work hard, to abstain from prestigious but unprofitable business and to take (only) risks which are worthwhile. In this setting one can derive the optimal combination of exogenously given financial instruments, usually debt and equity, but it appears difficult to characterize optimal financial contracts without imposing strong restrictions on stochastic properties of returns.[2] In the literature on security design in contrast, the manager is typically allowed to appropriate any cash–flow at no cost but investors may actively interfere

[2] If the issue of risk–incentive is left aside, as it is in this paper, and the 'monotone likelihood ratio property' applies, then the income of the manager is non–decreasing in returns. Payments to investors, however, will often decrease. Effort–incentive contracts for risk–averse agents are highly sensitive to the stochastic properties of returns; Holmström (1979) and Hart & Holmström (1986). When managers are risk–neutral but wealth–constrained the investors obtain everything when returns are low and receive nothing when returns are high; Innes (1990).

to prevent this from happening, a feature introduced by Townsend (1979) under the notion of 'costly state verification' (CSV).[3] In line with Jensen's (1986) free–cash–flow–hypothesis (FCF), the management cannot be forced to pay out cash which lacks profitable investment opportunities within the firm, except if threatened with bankruptcy. Since standard debt is the optimal financial instrument in the CVS–FCF–setting, it provided a handy building block for many studies of security design and corporate finance, e.g. cost of bankruptcy, Hart & Moore (1989); financial intermediation, Diamond (1984); credit rationing, Williamsen (1987); benefits of collateral; Bester (1994) term–structure and seniority of debt, Berglöf & von Thadden (1994); leverage and investment, Hart & Moore (1995). But there are drawbacks. Notably, the approach is difficult to reconcile with (outside) equity–financing. [4] Often the manager's compensation takes the form of a pure bonus–contract and there is no inefficient appropriation of resources whatsoever at the optimal contract.

This paper links both strains of the literature by allowing financiers to impose costly constraints on the management in order to discourage appropriation. By reinterpreting 'costly state verification' as 'costly curtailment of managerial control' and by explicitly modeling the dependence of appropriation cost on external interference, we obtain a fairly simple, one–period principal agent model. In its focus on control and incentives the paper is related to recent contributions on security design and corporate governance e.g. Zender (1991), Aghion & Bolton (1992), and Dewatripont & Tirole (1994). In Zender (1991) limited wealth prevents a single investor from becoming the residual claimant in all states. Since both investors are equally capable of managing the project,

[3]Contrary to what this label suggests the formal analysis does not depend on the true state being actually revealed. If this would be the case, payments could be made contingent on both, the manager's report and the result of the verification process. This would typically lead to stochastic auditing combined with some sort of 'punishment' if caught lying and a 'reward' for truth–telling. These features, however, are absent in this strain of literature — as they are in corporate finance.

[4]Berglöf & von Thadden (1994) introduce a contractible asset, which supports the claims of equity holders. In Hart & Moore (1995) the problem vanishes in some final period. In both cases the value of equity is given by the present value of assets, which shareholders can seize when the firm stops operating. Since modern corporations tend to stop operating only in case of bankruptcy, such a figure should not be much different from zero.

a debt like contract, may achieve the first best allocation under special circumstances. In a similar argument Aghion & Bolton (1992) derive the optimality of debt as a means to allocate control rights. The present paper is closest in spirit to Dewatripont & Tirole (1994) who derive debtholder activism and shareholder passivity as features of optimal contracts in an effort–incentive setting. It differs, however, in its emphasis on the relation between pay–related incentives and external interference and in its attempt to assess the robustness of some results by considering the role of risk–preferences, allowing for richer strategy–spaces and more general state–spaces.

With respect to investment, agency models of corporate finance have produced conflicting results. According to Jensen (1986) hard, debt like claims are needed to curb managerial overinvestment in the presence of free–cash–flow. However, if cash flow is poor, a 'debt overhang' may result in too little investment (Myers 1977). Hart & Moore (1995) and Berkovitch & Kim (1990) derive the amount and the structure of debt (chosen ex–ante) from a detailed analysis of the under/overinvestment trade off ex–post. Here, we derive optimal financial instruments from rather general assumptions about the nature of the ex–post incentive problem and ask how ex–ante investment is distorted.

Overview. In section 2 we develop the framework for the analysis of financial contracts which delegate limited control–rights and provide for monetary incentives. In section 3 we focus on constraints of managerial control and consider a setup which keeps us closed to the above mentioned literature on security design. The optimal financial contract turns out to be a combination of debt and equity, implying financiers activism when performance is poor while implementing the first best behaviour (zero appropriation) on part of the manager, in all states. In section 4, the emphasize is on how appropriation is distorted away from first best. Assuming convex appropriation cost, we explain why optimal contracts exacerbate 'bad luck' through poor incentives and motivate the use of option–plans in compensation schemes. Section 5 concludes. To streamline the exposition all proofs have been relegated to a technical appendix.

2 The Framework

We consider a manager, possibly risk averse, with access to an investment opportunity generating an uncertain return of value $\theta \in [\underline{\theta}, \bar{\theta}] \equiv \Theta, \underline{\theta} \geq 0$ with cumulative probability distribution F. Lacking initial wealth, she needs to raise funds from risk–neutral financiers, also referred to as investors. In compensation, these obtain a 'share' $s(\cdot)$ of the project's return. Following the free–cash–flow hypothesis we assume that θ is non–contractible. As the manager learns θ, by virtue of the tasks she is in charge of, she may 'appropriate' part of it, leaving only $x \geq 0$ to be shared.[5] Since we are suggesting a general framework, we stay deliberately vague about how potential firm value θ is turned into realized firm value x. It may be anything from fraud to empire building. Accordingly, there are many ways in which 'official' income, $x - s(\cdot)$, and appropriated cash–flow, $\theta - x$, may enter the manager's utility U. In the standard FCF–CSV–setting both are simply lumped together, implying that the manager considers money on the firm's account as good as her private wealth. This, however, appears to be highly implausible in most applications. In the corporate setting, for example, appropriation may take the form of continuing unprofitable lines of business to avoid conflict with subordinates. The profits foregone may easily be in the range of millions of dollars. In most cases the manager would not be prepared to spend a modest fraction of this amount, if it were her own income. Here, we introduce appropriation cost $h(\theta - x, \cdot)$, measured by the difference between real resource cost of 'inofficial' consumption and the 'official' income which would leave the manager equally well off. Her consumption is then given by: $\theta - s - h(\theta - x, \cdot)$. Throughout, it is assumed that h is nondecreasing in $|\theta - x|$, $h = 0$ for $x = \theta$ and $h \geq 0$ otherwise.

Intuitively, the higher appropriation cost are, the less inclined towards appropriation the manager will be. Such an alignment of interests may reflect personal attitudes, e.g. a strong preference for straight pay over perks, tolerance for organizational slack etc. It may also be a result of circumstances.

[5] We do not rule out $x > \theta$. In some cases 'faking' good outcomes may be possible, although it will not happen at the optimal contract.

In a growing industry, an 'empire building' visionary may pursue the strategy which is also favoured by investors. Often, however, interests will diverge and financiers may be obliged to impose constraints on managerial control in order to increase appropriation cost. In the following we consider general constraints, the tightness of which is measured by $\alpha \in [0, 1]$, and state contingent interferences, denoted $\beta \in [1, \bar{\beta}]$. Examples of the former are constraints imposed by the corporate charter, organizational design, accounting and publicity rules, restrictions on ownership stakes with business–partners etc. Examples for financiers' action which depend on performance are the imposition of covenants restricting capital expenditures or requiring the sale of assets, external evaluation, active intervention by investors or the replacement of the incumbent management.

Curtailment of managerial control, however, shall not circumvent the underlying agency–problem. Beside making appropriation less attractive, α and β result in 'constraint cost', $m(\alpha)$ respectively $v(\beta)$. For convenience it is assumed that these cost are born by the investor whose payoff is given as: $s(\cdot) - v(\beta) - m(\alpha)$. Depending on the nature of the constraints various interpretations of these cost are possible. Obviously, m includes some cost of monitoring and accounting to prevent theft. It will, therefore, be referred to as 'monitoring cost' which, however, should not be taken too literally. When a new project is separated from existing business to prevent cross–subsidization, the cost may consist of foregone economies of scale and scope. If investors actively interfere with decision–making or take over control, v indicates the comparative advantage of the manager to run the company. It may also stand for lost opportunities, if covenants force the management to abstain from certain activities or limit capital expenditures and for price discounts if financiers coerce the selling of assets. Finally, v may include the cost of verifying θ to prevent appropriation as in Townsend (1979).

All along it will be assumed that $m(0) = 0, m' > 0, m'' > 0, \lim_{\alpha \to 1} m = \infty$ and $v(1) = 0, v' > 0, v'' \geq 0$. Sometimes it will be necessary to restrict the class of distribution functions to ensure decreasing marginal returns in a probabilistic sense.[6]

[6]More precisely: log–concavity of $(1 - F)$ is equivalent to: $E[\theta | \theta \geq \hat{\theta}] - \hat{\theta}$ is decreasing in

Assumption 1 *: $1-F$ is log–concave; or equivalently the hazard rate is non–decreasing: $d(f/(1-F))/d\theta \geq 0$.*

For given appropriation cost $h(\theta - x, \beta, \alpha)$, constraint cost $v(\beta), m(\alpha)$ and distribution F, the financial contract optimally stipulates general and state contingent constraints, α respectively β, and payment s, taking into account their impact on the choice of x. Eventually we want to relate observable state contingent interference $\beta(\cdot)$ and payments $s(\cdot)$ to observable performance x. Hence, we consider the following timing of moves:

1. contract determines α, β, s,
2. α is implemented,
3. the manager observes θ and announces x,
4. β is implemented, the announced x is chosen by the manager and shared according to s.

With these assumptions we obtain a fairly basic principal agent problem with the agent making an informed choice. The optimal contract solves:[7]

Program 0.

$$\max_{s(x),\beta(x),\alpha,x(\theta)} \int_\Theta U[\theta - s(x) - h(\theta - x, \beta(x), \alpha)]\, dF \tag{P.0}$$

s.t.

$$\int_\Theta [s(x) - v(\beta(x)) - m(\alpha)]\, dF \geq I \tag{PC.0}$$

$\hat{\theta}$. See Bagnoli & Bergstrom (1989) for details. The assumption is needed for second order conditions and comparative statics.

[7]The problem has been stated in its most natural form, with the sharing rule and control schedule depending on the observable variable x, which is selected according to an incentive compatibility condition. Technically, the solution to program 0 is optimal among the class of 'indirect incentive compatible mechanisms'. Under the specific assumptions considered in the next section, this is more convenient and without loss of generality. In section 4 we will imply the 'revelation principle' and analyze the corresponding 'direct' mechanism.

$$x(\theta) = \arg\max_{\tilde{x}} U[\theta - s(\tilde{x}) - h(\theta - \tilde{x}, \beta(\tilde{x}), \alpha)] \qquad \forall\, \theta, x \tag{IC.0}$$

$$s(x) \leq \theta - h(\theta - x, \beta(x), \alpha) \qquad \forall\, \theta, x \tag{WC.0}$$

$$0 \leq x; \tag{NN.0}$$

Pareto–efficiency requires that the manager's expected utility is being maximized subject to the constraint that the investor is not made worse off. The participation constraint (PC.0) ensures that the investor's expected payoff is not less than some exogenous level I. In a perfectly competitive setting the opportunity cost of the investment would provide for such a lower bound. The market for managerial inputs imposes a parallel restriction on the manager's expected utility which has been dropped for convenience. Throughout the following it is assumed that a solution satisfying both reservation utilities exits.[8]

If θ were contractible, managerial control would always be unrestricted ($\alpha = 0, \beta = 1$) to minimize 'constraint cost' $v + m$, and her consumption would only consist of straight pay $x = \theta$, to avoid appropriation cost. The sharing rule $s(\cdot)$ could be used freely to satisfy the reservation utilities of the two sides and to provide for insurance in case of risk aversion. With θ being not contractible, however, the manager is free to choose x, given α, β, s and θ, in order to maximize her utility as stated in the incentive constraint (IC.0).[9] By making the manager residual claimant in all states ($s(x) = \bar{s}$), the first best choice of α, β and x could still be implemented. This, however, would not only require her to bear all income–risk. But provided that I is high enough, hence $\bar{s}$ large enough, it will also conflict with the wealth constraint (WC.0) which prevents the ex–post consumption of the manager from falling below zero.[10]

[8] The same features of efficient contracts could be obtained by maximizing the investor's expected profits subject to a suitable reservation utility for the manager. Since I is arbitrary, the particular setup does not make an assumption about the bargaining power of the two sides — contrary to what is sometimes claimed in the literature.

[9] To avoid technical complexities, we assume that x is unique. In case of being indifferent, the manager shall select the value closest to θ.

[10] Following Sappington's (1983) seminal paper, this constraint is often referred to as 'limited liability' in the principal agent literature. In finance, 'limited liability' refers to a feature of the financial contract which ought to be explained not assumed. Since we do not

Finally, the non–negativity constraint for x, imposes a limit on the amount of appropriation $\theta - x$.

3 Proportional Appropriation Cost with two Control Regimes

In this section we consider a variant of the basic setting which keeps us closed to the established literature on security design. We restrict state contingent intervention β to takes two values only: $\beta \in \{1, \bar{\beta}\}$. For $\beta = 1$ financiers remain passive and appropriation cost shall be proportional to the amount of resources diverted to managerial consumption. The constant factor being given by the intensity of non–contingent constraints, α.[11] If financiers become active, $\beta = \bar{\beta}$, appropriation cost shall raise to infinity.

Assumption 2

$$h(\theta - x, \beta, \alpha) = \begin{cases} \alpha|\theta - x| & \text{if} \quad \beta = 1 \\ \infty & \text{if} \quad \beta = \bar{\beta}, x \neq \theta \\ 0 & \text{else} \end{cases}$$

From a substantive point of view this setup appears quite restrictive. The appropriation loss would be proportional if the manager diverts part of the output to a black market at a discount–price — which is not very convincing in the corporate setting, we have in mind. It might, however, be rationalized by a more developed model which allows for inter–temporal smoothing of appropriation in the spirit of Holmström & Milgrom (1987). The 'bang–bang'–nature of constraints on managerial control is clearly at odds with the

assume 'limited liability' with respect to securities (nor do we explain it), the notion 'wealth constraint' is used to avoid misconceptions. Being a real cost, h does not allow the contract to cut through the manager's wealth constraint. This is a difference to Diamond (1984) in which the wealth constraint is overcome through non–pecuniary penalties.

[11] To simplify notation, we assume the same cost for faking. This does not effect the results.

variety of measures employed when firms run into financial distress. Analytically, however, these assumptions provide a good starting point. Obviously, infinite appropriation cost for $\bar{\beta}$ corresponds to the case of 'verification' in the CSV–setting and proportionality includes as a limiting case ($\alpha = 0$), the assumption of costless appropriation which has been adopted in much of the literature.

In this particular setting, the financial contract, is composed of a sharing rule $s(x)$, a set A (the complement is P) for which investors take action to curb managerial control ($\beta = \bar{\beta}$), and the intensity of non–contingent accounting and monitoring procedures α. First, we translate the incentive constraint. Since appropriation cost are infinite for $x \in A, x \neq \theta$, the manager will never choose $x \in A$, except for $\theta = x$. As in the CSV–literature, interference by financiers prevents any appropriation. If financiers remain passive, the incentive constraint imply the following restrictions on the slope of the payment–schedule s.

$$s(x) - s(x_0) \leq \alpha(x - x_0), \quad \forall\, x_0 \in P \tag{IC.a}$$

$$s(x) - s(x_0) \leq -\alpha(x - x_0), \quad \forall\, x_0 \in P \tag{IC.b}$$

If (IC.a) is violated, the manager would prefer to appropriate the difference $x - x_0$ whenever $\theta > x$. If (IC.b) is violated, she would prefer to fake the difference for $\theta < x_0$. Linearity of these constraints obviously implies that managerial behaviour will not deviate from first best:

Proposition 1 *If appropriation cost are proportional, optimal appropriation is zero: Assumption 2* $\Longrightarrow x(\theta) = \theta, \forall\theta$.

Anticipating that (IC.b) will not be binding we are left with the following problem:

Program 1.

$$\max_{s(x),A,\alpha} \int_{\Theta} U[\theta - s(x)]\, dF \tag{P.1}$$

s.t.

$$\int_\Theta s(x)\,dF - \int_A v(\bar\beta)\,dF - m(\alpha) \geq I \qquad \text{(PC.1)}$$

$$s(x) \leq s(\tilde x) + \alpha(x-\tilde x), \quad \forall\, \tilde x \in P \qquad \text{(IC.1)}$$

$$s(x) \leq \theta \qquad \text{(WC.1)}$$

In the following subsections we consider solutions of this program for a risk neutral manager with a binding wealth constraint and a risk averse manager, the wealth constraint being slack.

Risk Neutrality

To begin with, we characterize the solution for an arbitrary α. Let $I^0 = \int_\Theta (\alpha\theta + (1-\alpha)\underline{\theta})\,dF$ denote the highest expected payout which, given α, can be achieved with no action taken in any state.

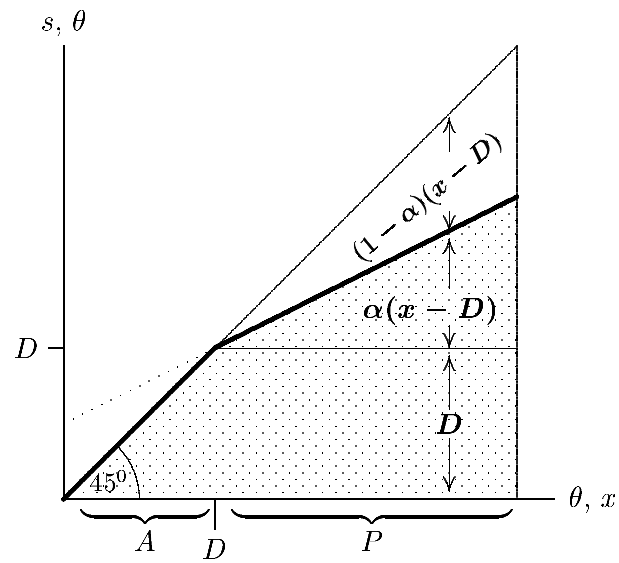
PROPOSITION 2 *Provided that appropriation cost are proportional, the manager is risk–neutral and the opportunity cost of the project is high enough, then there is a threshold value $D > \underline{\theta}$ below of which investors take action. When active, investors obtain the whole output. When passive, they receive fixed payment equal to D plus a share α of the residual:*
Assumptions 2, $U'' \equiv 0$ and $I > I^0 \Longrightarrow$ the solution to program 0, $\{A^*, s^*\}$ is of the form

$$\begin{aligned} A^* &= [\underline{\theta}, D); \quad D > \underline{\theta} \\ s^*(x) &= \begin{cases} x; & x \in A^* \\ D + \alpha(x-D); & x \in P^* \end{cases} \end{aligned}$$

A typical proposition 2 contract is illustrated in figure 1. The thick line illustrates s^*, the shaded area marks payoff to the investor and the blank area below the 45^0–line indicates the compensation for the manager. Obviously, proposition 2 includes pure debt–financing, as derived in Townsend (1979) and Gale & Hellwig (1986), as a limiting case for $\alpha = 0$.

Three features of this contract invite the interpretation in terms of risky debt and equity: (i) financiers take action on a lower interval and only if a fixed,

Figure 1: Optimal Financing with Proportional Appropriation Cost and Risk–Neutrality



hence debt–like, claim cannot be served. (ii) When active, they recover as much as possible. 'Taking action' may be interpreted as forcing the firm into bankruptcy with the creditors gaining control and laying the manager off. (iii) In good states investors remain passive. They obtain a fixed sum and a constant, hence equity–like, share α of the residual.

The intuition for the optimality of debt and equity thus stands as follows: For any given level of monitoring intensity α, the contract should minimize the cost of intervention by financiers, subject to the fulfillment of the participation constraint. If I would be very low, these cost could be avoided altogether, but nothing more specific can be said about the optimal contract. As I increases, the manager would issue state contingent, equity–like, claims and/or riskless debt until both sources are exhausted, i.e. when $s = \alpha x + (1 - \alpha)\underline{\theta}$. Since s cannot be made steeper without violating the incentive constraint, and cannot be shifted upward without violating the wealth constraint, no more financing could be obtained from passive investors. To increase expected payoffs investors have to become active in order to prevent the manager from diverting resources to her own ends. The cost of doing so are minimized by paying out as much as possible given the wealth constraint in bad states, respectively the incentive constraint in good states. As a result, the manager receives an information rent only in good states. Overall, her consumption is convex in θ.

As mentioned in the introduction, there is nothing in the model to suggest that s should be divided into distinct claims, debt and equity, or that payment in the control region, accrues exclusively to debt–holders. However, as financiers actively interfere with managerial control, they effectively become managers of the project and the incentive problem is set up anew. Those investors who resume control when returns are poor should, therefore, be the residual claimants. This suggest that payment in case of bankruptcy, accrues exclusively to one group of investors, i.e. debt–holders.[12]

So far we have established conditions for which optimal securities resemble a combination of debt and equity. However, with α exogenously fixed, the optimal contract maintains a fixed relation between inside and outside equity. As a result risky debt is the marginal source of finance and the capital structure, the relation of debt to outside equity, is trivially determined by the amount of external finance needed. While this may be an appropriate description of how firms deal with most of their daily financial decisions, sometimes firms do deliberately change the relation between inside and outside equity e.g when going public or undergoing a leveraged buyout. To address the optimal choice of α we use propositions 1 and 2 and drop unnecessary constraints to obtain:

PROGRAM 2.

$$\max_{\alpha,D} \int_{D}^{\bar{\theta}} (\theta - D)(1-\alpha)\, dF$$

s.t.

$$\int_{\underline{\theta}}^{D} (\theta - v(\bar{\beta}))\, dF + \int_{D}^{\bar{\theta}} (D + \alpha(\theta - D))\, dF - m(\alpha) \geq I \qquad \text{(PC.2)}$$

Since border solutions provide little insight we consider only interior solutions.

[12]See Aghion & Bolton (1992) and Zender (1991) for more developed analysis of this idea.

PROPOSITION 3 *Optimal financing balances marginal cost of monitoring, with marginal 'bankruptcy' cost, adjusted by a factor reflecting the relative importance of debt and equity–like claims in the solvency region. Convexity of* m *and log–concavity of* $1-F$*, however, are not sufficient to ensure uniquness of the optimal capital structure.*
Assumptions 1, 2 and $U'' \equiv 0$ *imply:*
(i) An interior solution of program 2 balances marginal cost according to:

$$v(\bar{\beta})f(D)\frac{E[(x-D)|x \geq D]}{1-\alpha} = m'(\alpha), \tag{1}$$

(ii) The solution is unique if:

$$\frac{m''}{m'} + \frac{m'}{vf}\left(f + (1-F)\frac{f'}{f}\right) > \frac{2}{1-\alpha}$$

The possibility of multiple local optima in this apparently simple setting may be considered disheartening. Imposing further restrictions on m and F to ensure uniqueness, however, is difficult to justify. It would also blur an important trade–off in the choice of the optimal capital structure. In terms of payoff for financiers, the marginal gains from increased monitoring are lower if the level of debt is higher. Not only is the likelihood of bankruptcy larger, in which case the cost of monitoring turns out to be wasted. A larger fixed claim also reduces the total value of the variable claim of which outside equity is only a fraction. When managerial control can be curbed through general constraints and through state contingent interferences, then there is a reason to rely more strongly on either one of these instruments and finance accordingly more equity–loaded, respectively, debt–loaded. This may explain why similar firms choose different capital structures and how minor changes in the environment may trigger major revisions in the financial structure.

Risk Aversion

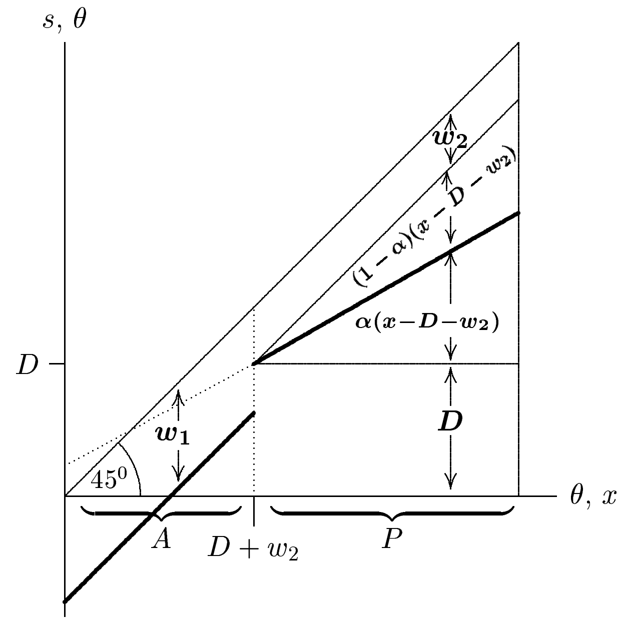
The manager's human capital is often idiosyncratic to the firm she is in charge of. Putting, risk–considerations aside, her financial assets should also be tied up in the firm due to the agency cost of external finance. Hence, risk neutrality cannot be justified by diversification. As an assumption about preferences, however, it is not convincing. Furthermore, in the principal–agent framework the design of financial securities cannot be disentangled from the compensation of the management. Proposition 2 predicts that the latter takes the particular form of a pure bonus contract, equivalent to a fraction $(1-\alpha)$ of total shares. Empirically however, top–managers' compensation schemes usually include substantial non–contingent base salaries. Risk–aversion may be one possible explanation for this feature. The following propositions reveals, however, that risk–aversion does not square well with the interpretation in terms of financial securities.

PROPOSITION 4 *With proportional appropriation cost and risk–aversion, investor and manager will obtain fixed payments (D and w_2 respectively) and constant shares of the residual (α and $1-\alpha$ respectively) when investors remain passive. As before, investors interfere only if the fixed claim cannot be met. In this case the investor becomes the residual claimant and the manager obtains a fixed fee w_1 which is larger than w_2 provided the latter is larger than zero:*
Ass. 2 $\Longrightarrow$ the solution to program 1, $\{A^*, s^*\}$ is of the form

$$\begin{aligned} A^* &= \begin{cases} [\underline{\theta}, D+w_2); & D+w_2 > \underline{\theta} \\ \emptyset; & \textit{otherwise} \end{cases} \\ s^*(x) &= \begin{cases} x - w_1; & x \in A^* \\ D + \alpha(x - D - w_2); & x \in P^* \end{cases} \\ \textit{either} \quad & w_1 \geq w_2 = 0 \\ \textit{or} \quad & w_1 > w_2 > 0 \end{aligned}$$

A typical proposition 4 contract is illustrated in figure 2. With risk–aversion, first best insurance would leave the manager with fixed consumption across all states, implying $s' = 1$. If investors remain passive, moving along the incentive constraint brings s as close as possible to this. Any other sharing rule would yield a stochastic consumption which is a mean preserving spread compared to a suitably chosen schedule with slope α. Once investors become active, the incentive constraint is essentially removed, and first best insurance can be achieved with no additional cost.

Figure 2: Proportional Appropriation Cost and Risk–Aversion



The trade–off between insurance and cost of interference determines the base salaries. In good states the manager will receive a base salary w_2, in addition to the bonus contract (or her equity share) — which is just what we are looking for. Unfortunately, she will receive an even larger fixed salary w_1 in states with returns low enough to trigger 'bankruptcy'. This implies that she is strictly worse off in some 'solvent' states compared to 'bankruptcy'. The intuition is the following. If her income would always be higher in the solvency–region, her expected marginal utility would be lower, if taken conditional on being solvent rather than on being bankrupt. Hence, by increasing w_1 while decreasing w_2 to keep investors' expected payoff constant, insurance could be improved without any change in bankruptcy cost.[13]

[13]Clearly, the discontinuity at D, which is also mentioned by Gale & Hellwig (1986) depends on appropriation cost being infinit for $\bar{\beta}$. However, for managerial consumption to decrease on some range, it suffice to have α being raised above unity. What matters is that income can be distributed from P to A at zero marginal cost.

4 Convex Appropriation Cost and Flexible Interference

While being more general than zero cost of appropriation, the assumption of proportional cost invites further generalization. Also, we know from proposition 1 that non–linear cost are a necessary condition for optimal contracts to display positive appropriation. Presumably, 'on the job consumption' through 'empire–building', perks and pet–projects suffers from decreasing marginal utility. Hence, we assume appropriation cost to be convex.[14] While the focus of this section is on appropriation, we do also consider state contingent intervention by financiers. In this respect we drop the restriction, that investors either remain passive or completely prevent appropriation and allow them to carefully dose the intensity of interference, $\beta \in [1, \bar{\beta}]$. Throughout, however, α is considered as fixed and suppressed in the notation. Formally, we replace assumption 2 by:

Assumption 3
$h(\theta - x, \beta, \alpha) = a(|\theta - x|)\beta$ *with* $0 < a' < 1, \quad a'' \geq 0, \quad a''' \leq 0 \;\; \forall x \neq \theta$

The multiplicative structure is chosen for technical convenience. It allows us to discuss the issues of appropriation and interference rather independently. $a' < 1$ is needed to obtain a meaningful incentive problem. $a''' \leq 0$ implies that a''/a' is decreasing and needed to ensure that first order conditions are sufficient. In the previous section optimal contracts turned out to be piecewise continuous. Now they are assumed to be of this class and program 0 is transformed into a problem amendable to optimal control theory.[15]

As stated the problem is not differentiable, since a is kinked at $x = \theta$. However, as it is to be expected from the discussion of the previous sections the

[14]However, the prestige associated with size may also exhibit some economies of scale. Arguably becoming 'the number one' yields rewards which are disproportionately larger than the same increase in size further down the hierarchy.

[15]The approach is an adapted from Guesnerie & Laffont (1984) and Laffont (1989) and fairly standard in screening models.

optimal contract will not feature faking. Formally, we introduce a non–faking–constraint (NF): $0 \leq \theta - x(\theta)$ and consider the right hand derivatives of a at zero only. Implying the *revelation principle*, we rewrite the contracting problem as a *direct* incentive compatible mechanism. The contract consists of $s(\theta)$, $ß(\theta)$ and $x(\theta)$, all depending directly on the unobservable variable as signaled by manager. The incentive constraint, becomes a 'truth–telling–constraint', requiring that, given s, $ß$ and x and it is in the interest of the agent to convey the true θ.

$$\theta = \arg\max_{\tilde{\theta} \in \Theta} U(\theta - s(\tilde{\theta}) - a(\theta - x(\tilde{\theta}))ß(\tilde{\theta})) \qquad \forall\, \theta \in \Theta \tag{IC}$$

Next we substitute for $s(\theta)$ and transform the incentive constraint. Let c denote the manager's consumption when telling the truth

$$c(\theta) \equiv \theta - s(\theta) - a(\theta - x(\theta))ß(\theta). \tag{2}$$

In addition, define his consumption when lying

$$\tilde{c}(\theta, \tilde{\theta}) = \theta - s(\tilde{\theta}) - a(\theta - x(\tilde{\theta}))ß(\tilde{\theta})$$

where $\tilde{\theta}$ denotes his message, possibly false. Obviously $c(\theta) \equiv \tilde{c}(\theta, \theta)$, hence $c'(\theta) = \tilde{c}_{\theta}(\theta, \theta) + \tilde{c}_{\tilde{\theta}}(\theta, \theta)$. Defining $g(\theta) \equiv \tilde{c}_{\tilde{\theta}}(\theta, \theta)$ we obtain:

$$c'(\theta) = 1 - a'(\theta - x(\theta))ß(\theta) + g(\theta) \tag{ICa}$$

Rewrite the incentive constraint as:

$$\theta = \arg\max U(\tilde{c}(\theta, \tilde{\theta})) \quad \text{s.t.} \;\; 0 \leq x(\theta); \quad 0 \leq \theta - x(\theta) \qquad \forall\, \theta \in \Theta$$

and consider an interval $\Theta_1 \subseteq \Theta$ for which $0 < x < \theta$. The first order condition for truth–telling being the optimal lie is $\tilde{c}_{\tilde{\theta}}(\theta, \theta) = -s' + a'ßx' - aß' = 0$, or equivalently $g(\theta) = 0$. The second order condition requires $\tilde{c}_{\tilde{\theta}\tilde{\theta}}(\theta, \theta) < 0$. Since the first order condition assumes the status of an identity on Θ_1 we have $\frac{d}{d\theta}\tilde{c}_{\tilde{\theta}}(\theta, \theta) = \tilde{c}_{\tilde{\theta}\tilde{\theta}}(\theta, \theta) + \tilde{c}_{\tilde{\theta}\theta}(\theta, \theta) = 0$. Hence, the second order condition requires $\tilde{c}_{\tilde{\theta}\theta}(\theta, \theta) \geq 0$, or

$$0 \leq a''(\theta - x(\theta))x'(\theta) - a'(\theta - x(\theta))ß'(\theta) \tag{M}$$

which is fulfilled if $x' \geq 0$ and $ß' \leq 0$ (monotonicity constraint). Now consider an interval $\Theta_2 \subseteq \Theta$ on which (NF) binds, i.e. for which $\theta = x$. It is necessary and sufficient to have $\tilde{c}_{\tilde{\theta}}(\theta,\theta) \geq 0$, or equivalently $g(\theta) \geq 0$. Given (NF), we therefore may replace the incentive constraint by (ICa), (M) and

$$0 \leq g(\theta), \qquad 0 = (\theta - x(\theta))g(\theta) \tag{ICb–c}$$

Finally, we translate the wealth constraint. Expressed in terms of consumption it requires $c(\theta) \geq \theta - a(\theta - x(\theta))ß(\theta) \equiv \underline{c}(\theta)$. Provided the monotonicity constraints are satisfied (ICa) implies $c' \geq \underline{c}'$. Hence, it is sufficient to ensure that the constraint is met at the lower bound. Summarizing all these steps we obtain the following control problem with state variable c and control variable x, $ß$ and g.

Program 3.

$$\max_{c(\theta),x(\theta),ß(\theta),g(\theta)} \int_\Theta U(c(\theta))\,dF \tag{P.3}$$

s.t.

$$0 \leq \int_\Theta [\theta - v(\beta) - c(\theta) - a(\theta - x(\theta))ß(\theta)]\,dF - I \tag{PC.3}$$

$$c'(\theta) = 1 - a'(\theta - x(\theta))ß(\theta) + g(\theta) \tag{ICa.3}$$

$$0 = (\theta - x(\theta))g(\theta) \tag{ICb.3}$$

$$0 \leq g(\theta) \tag{ICc.3}$$

$$0 \leq a''(\theta - x(\theta))x'(\theta) - a'(\theta - x(\theta))ß'(\theta). \tag{M.3}$$

$$0 \leq \theta - x(\theta) \qquad \forall\,\theta \in \Theta \tag{NF.3}$$

$$0 \leq x(\theta) \qquad \forall\,\theta \in \Theta \tag{NN.3}$$

and boundary condition

$$c(\underline{\theta}) \geq \underline{\theta} - a(\underline{\theta} - x(\underline{\theta}))ß(\underline{\theta}), \qquad c(\bar{\theta}) \quad \text{free} \tag{B.3}$$

In analogy to the previous section let $I^0 = \int_\Theta (a'(0)\theta + (1 - a'(0))\underline{\theta})\,dF$ denote the maximal expected payout of a no–distortion contract. The multiplier associated with PC3 is λ. In the following we discuss the solution to this problem for some special cases.

Risk–Neutrality

We begin with a rather complete characterization of the optimal contract for a risk–neutral manager when curtailment of control is exogenously fixed to its minimum ß$(\theta) = 1$.

Proposition 5 *With risk–neutrality, minimal constraints and required payout sufficiently high, optimal appropriation is positive in bad states and strictly decreasing or at its maximum.*
Assumptions 1, 3, $U'' = 0$, ß $= 1$ and $I > I^0 \implies$ a solution to program 3 is of the form:

$$\left.\begin{aligned} x &= 0 \\ \mathsf{s} &= 0 \end{aligned}\right\} \quad \textit{for} \quad \theta \in [\underline{\theta}, \max\{\check{\theta}, \underline{\theta}\}]$$

$$\left.\begin{aligned} x < \theta,\ x' &> 1 \\ \mathsf{s}'(\theta) &= a'(\theta - x) \cdot x' \end{aligned}\right\} \quad \textit{for} \quad \theta \in (\max\{\check{\theta}, \underline{\theta}\}, \hat{\theta})$$

$$\left.\begin{aligned} x &= \theta \\ \mathsf{s}'(\theta) &= a'(0) \end{aligned}\right\} \quad \textit{for} \quad \theta \in (\hat{\theta}, \bar{\theta}]$$

on $[\check{\theta}, \hat{\theta}]$ x is implicitly defined by

$$f(\theta)a'(\theta - x)\lambda = (1 - F(\theta))a''(\theta - x)(\lambda - U') \tag{3}$$

where $\check{\theta}$ and $\hat{\theta}$ solve (3) for $x = 0$, respectively $\theta - x = 0$.

The contract is shaped by the following trade–off: High payouts are needed in order to fulfill the participation constraint i.e. information rents have to be kept low. Accoring to the incentive constraint this can only be achieved by tolerating appropriation. Suppose we increase appropriation at θ marginally holding the utility of the manager constant. The expected loss would be fa' times the shadow price of funds, λ. By the incentive constraint (ICa) this would allow for an increase of payouts by a'' not just for θ but for all better realizations, in probability $(1-F)$. The net–value of these transfers is $(\lambda - U')$, the difference between shadow value of funding and marginal utility of managerial income. Equation (3) states that x will be chosen to equate marginal cost of appropriation to marginal benefits of the resulting increase of payouts. It can be rewritten as

$$\frac{f(\theta)}{1-F(\theta)} = \frac{a''(\theta - x)}{a'(\theta - x)}(1 - \frac{U'}{\lambda})$$

Assumption (3) implies that the right hand side is decreasing in x for any given θ. By log–concavity of $(1 - F)$ (assumption 1) the left side decreases monotone towards zero. The higher the returns, the less likely it is to receive an even higher return. The gains from distorting x away from the first best level shrink towards zero as θ approaches the top of its distribution. Since marginal appropriation cost are always positive, there is no distortion at the top of the distribution.

The claims in proposition 5 refer to variables which are not observable. In order to compare the results with those of the preceding sections, we restate them in terms of $s(x)$:

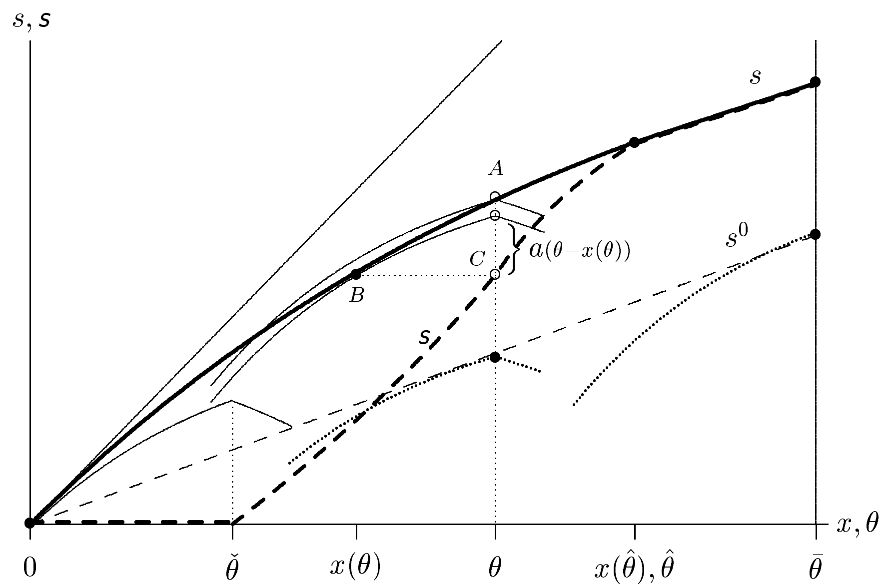
COROLLARY 1 *Payments to both sides are strictly increasing in realized returns, those to the manager (investor) are convex (concave):*
$\exists\ \hat{x} \in (\underline{\theta}, \bar{\theta})$ *such that:*

$$s'(x) = a'(\theta - x) > a'(0),\ s''(x) < 0 \quad \textit{for } x < \hat{x}$$
$$s'(x) = a'(0) \quad \textit{for } x > \hat{x}$$

The payment schedule resembles a compensation scheme which, in addition to shares, includes call options on the stock with different exercise prices, the highest of which is $\hat{\theta}$.[16] The optimal contract may feature a base–wage even in the case of risk–neutrality. However, this depends on the non–negativity constraint being not binding. Therefore, lowering $\underline{\theta}$ (more likely for small firms) or raising I (more likely for large firms) will work against a fixed base wage. It is also interesting to note that if $0 \leq x$ binds at the optimal contract, then the distribution of observable returns x will have a masspoint at its lower bound, even if the distribution of potential returns θ is atomless. If things go wrong, the value of the firm will be 'surprisingly' often close to zero, creating the impression that 'bankruptcy' tends to come too late. There is some evidence

[16] A similar result is obtained in Chiesa (1992).

Figure 3: Optimal Financing with Convex Appropriation Cost



for this feature in particular among small firms. In Germany for example more than three quarters of the bankruptcy petitions are dismissed for insufficient bankrupts' assets.

These features are illustrated in figure 3 displaying payments and state–contingent indifference curves in the x, s–plane. The thinly dashed line displays the highest payout s^0 to security holders compatible with zero appropriation, i.e. a slope of $a'(0)$ everywhere. The dotted lines show the corresponding indifference curves of the manager contingent on the realization of θ. They increase towards the south and are kinked at the point of zero appropriation because marginal appropriation is always costly. If the participation constraint cannot be fulfilled with the s^0 schedule, it has to be raised. The thick curve illustrates such a payout–schedule $s(x)$ which can only be achieved at the cost of inefficient appropriation. For high realizations, those to the right of $\hat{\theta}$, appropriation is still zero. However, to achieve these high payouts, the slope has to be increased for low values, to the left of $\hat{\theta}$, which requires a deviation from efficiency. If θ realizes and the manager would refrain from appropriation, he is left with $\theta - s(\theta)$ (point A). By appropriating the amount $\theta - x(\theta)$ he moves to point B, where he achieves the highest feasible utility, given s and θ. While his 'official' income is lower than in A, he is more than compensated through the utility obtained from appropriated cash–flow. The investor is left with $s(x(\theta))$, respectively $\mathsf{s}(\theta)$ which is given by the thickly dashed curve through C. The resource loss due to appropriation is indicated by the difference between the payment at zero–appropriation which would make the manager indifferent to B and the feasible payment at C. For good states effective payments are increased over what is feasible without appropriation. However for low values of θ, both sides are worse off.

State–Contingent Constraints on Managerial Control

The contract derived as optimal for risk neutral managers in proposition 5, implies low powered incentives in bad states and high powered incentives in good states. It is, therefore, to be expected that state–contingent constraints will be more intensively used in bad states to compensate for poor incentives. To keep the argument simple, we confine the analysis now to a range $\Theta_0 \subseteq \Theta$ on which the contract is characterized by first order conditions.

Proposition 6 *With risk–neutrality and flexible intervention the level of appropriation and the intensity of interference decrease and payments to investors increase in θ.*
Assumptions 1, 3, $U'' = 0 \Longrightarrow$ for $\theta \in \Theta_0$ the solution to program 3 entails:

$$x' > 1; \quad \text{ß}' < 0; \quad \mathsf{s}'(\theta) = a'\,\text{ß}x' - \text{ß}'a > 0$$

ß is implicitly defined by

$$f\lambda(v' + a) = (1 - F)a'(\lambda - U') \tag{4}$$

and x is given by (3)

Like appropriation, optimal intervention ß balance the local cost of increasing s, the left hand side of (4), against the net gains from higher payouts in all better states, right hand side of (4). By monotonicity of x we can restate the results in terms of observable variables:

Corollary 2 *Payment to financiers is increasing and interference from financiers is decreasing in observed performance:*

$$\beta'(x) < 0; \qquad s'(x) = a'\beta - \beta' a > 0$$

There is clear empirical evidence for such an inverse relation between curtailment of managerial control and observable returns. For example Kaplin (1994a) and Kaplin (1994b) find that management turnover and outside appointments on the supervisory board increase with poor earnings and stock performance. Interesting enough these correlations show up in comparable strength under the rather different system of corporate governance in the U.S.A and Germany / Japan. Similarly Gilson (1990) documents high rates of management turnover for financially distressed firms and a much increased use of covenants severely limiting the discretion over capital expenditures.[17]

Finally, note that the optimal contract in case of risk–neutrality may induce a similar preferences for risk taking as debt–financing in Jenson & Meckling (1976). Differentiating (ICa) shows:

Corollary 3 *The manager's consumption, through payment and appropriation, is convex in potential returns whenever the non–negativity constraint does not bind:*

$$c''(\theta) = -a'' \cdot (1 - x')\text{ß} - a'\text{ß}' > 0, \quad \forall \theta > \max\{\check{\theta}, \underline{\theta}\}$$

If $0 \leq x$ does not bind, the manager strictly prefers a mean preserving spread of the distribution F. She might, therefore, also prefer a an increase of risk which is accompanied by a small decrease of expected returns. If these opportunities exist, well known inefficiency due to costly 'asset–substitution' arise, and the contract would have to be modified accordingly.

[17] Unfortunately, with the added instrument ß, we cannot show that payout is necessarily concave, which would require $s''(x) = a''\beta(1/x' - 1) + a'\beta' - a'\beta'(1/x' - 1) - a\beta''$ to be negative. Since $1/x' < 1$ all terms have the right sign except the last which is ambiguous. We cannot offer intuitive conditions for the primitives (a, v, F) which ensure that $\beta'' > 0$ in this model.

Risk–Aversion

As became clear in the previous sections, the desire to insure the manager and the need to fulfill the participation constraint work into the same direction with respect to the slope of the optimal payment schedule: both make s steeper. Since the focus is on risk–preferences and on the role of appropriation we assume that the wealth constraint is not binding and set $\beta = 1$ again.[18] Let $\hat{x}$ solve program 3 when the monotonicity constraint is ignored, in the appendix it is shown that sign $\hat{x}'$ is equivalent to

$$\text{sign}\ \frac{d(f/F)/d\theta}{f/F} - \frac{d(a''/a')/d\theta}{a''/a'} - \frac{F}{f}\frac{a''}{a'}\frac{U'}{EU'} \tag{5}$$

where EU' denotes the expected marginal utility. In this expression only the second term has the desired sign (by assumption 3). The third term is negative and the first term is also negative if F is log–concave, which is often the case when $1 - F$ is log–concave.[19] Therefore, the solution will generally entail intervals on which the monotonicity constraint binds. Fortunately, the following partial characterization is possible without incorporating monotonicity explicitly into the program.

PROPOSITION 7 *With risk–aversion sufficiently strong and appropriation cost sufficiently convex optimal appropriation is positive for intermediate states. Assumptions 3 and $a''(0)/a'(0)$ large enough $\Longrightarrow$ a solution to program 3 is of the form: $\exists \Theta_m \neq \emptyset$, $\inf \Theta_m > \underline{\theta}$, $\sup \Theta_m < \bar{\theta}$ such that:*

$$\begin{aligned} x < \theta;\ \ s' = a'x' \quad & \textit{for } \theta \in \Theta_m \\ x = \theta;\ \ s' = a'(0) \quad & \textit{else} \end{aligned}$$

for $\theta \in \Theta_m$, and $x' \geq 0$ not binding, x is implicitly defined by

$$fEU'\frac{a'}{a''} = F(1-F)(EU'_{-} - EU'_{+}) \tag{6}$$

[18] Emphasizing conditions under which the optimal contract entails zero appropriation this problem is analyzed in Lacker & Weinberg (1989).

[19] Most common examples of distributions with a log–concave $(1-F)$ 'derive' this property from the log–concavity of their density functions f which is sufficient for log–concavity of both $(1 - F)$ and F (see Bagnoli & Bergström (1989) for details).

where $EU'_{-}(\theta) \equiv E[U'(c(\tau))|\tau \leq \theta]$, $EU'_{+}(\theta) \equiv E[U'(c(\tau))|\tau > \theta]$ *denote expected marginal utility conditional on the state being worse than* θ*, respectively better.*

The intuition for the distortion taking place in the middle of the distribution is to be gained from (6), the first order condition for an interior solution of x. On the right hand side we have the expected utility gain from a redistribution of a marginal amount of income from better states to worse states, keeping expected income constant. This gain will be larger, the more different the conditional marginal utilities are, i.e. the more the manager's consumption varies in θ. However, as we move towards the lower bound of the support, an increasing amount of income taken in 'better' states is in fact taken from states which provide for a consumption below average. Similarly, as we move towards the upper bound, an increasing amount of income paid out in 'worse' states, helps in states with consumption above average. Therefore the gains from improved risk–sharing approach zero at both boundaries of the distribution. On the left hand side we have the increase of appropriation cost (weighted with the marginal utility of income) required by the incentive constraint if the consumption schedule is to be smoothed at some particular point. This is always positive if $a'(0) > 0$ but can be made arbitrarily small by letting a become sufficiently convex.

Optimal Investment

So far, we argued that with non–contractible cash–flow, optimal financial contracts may feature costly distortions in form of constraints on managerial control and wasteful appropriation of cash–flow. Now, we ask how this relates to investment. Intuitively, one would expect agency–cost to reduce investment. The more the project depends on external finance, the less attractive it becomes due to costly curtailment of managerial control and the waste of resources in appropriation. In our partial analysis of optimal contracting the tax–like effect of expected agency–cost results in a shadow price of external finance λ which is larger than one. Compared to a first best world, in which the manager would be wealthy enough to realize the project on his own, there should be underinvestment. If we redistribute a dollar from the manager to the financier, the manager's utility loss is larger than a dollar if incentive constraints are binding. Hence, expected returns to managerial input are reduced, which will eventually violate his participation constraint — marginal projects become unfeasible. Therefore, it is often taken for granted that agency cost of external finance make investment excessively dependent on the availability of uncommitted cash–flow.

However the tax–analogy failes to account for the interaction between investment and agency cost in intramarginal projects. Since, ex post, distortions are concentrated at the bottom of the distribution, the 'agency–tax' is decreasing in returns. This creates an additional incentive to achieve higher returns through increased investment, provided the project is wortwhile ex–ante. Hence, for intramarginal projects a reduction of internal funds has two opposing effects. On the one hand, the increase of expected agency–cost makes external funding more costly, suggesting a reduction of investment. On the other hand, to the extend that the gap between payoffs (net of agency–cost) in good and bad states widens, it increases the expected returns to investment.

To address this issue we assume that investment increases expected returns at a decreasing rate. Technically, I becomes an instrument which shifts the probability distribution of returns, $F(\theta, I)$, in the sense of first order stochastic dominance, $F_I < 0$, $F_{II} > 0$. In order to streamline the exposition we consider only the case of risk–neutrality (normalizing $U' = 1$) and assume $a'(0) = 0$ which ensures that $x < \theta$ for all $\theta < \bar{\theta}$ whenever the incentive constraint binds, $(\lambda > 1)$. Finally, in this case the optimal contract shall also feature interiour solutions for x and ß.

Integration of the incentive constraint (ICa.3) gives managerial consumption as $c(\theta) = \int_{\underline{\theta}}^{\theta}(1 - a'ß)d\theta$. Partial integration of expected consumption $\int_{\Theta}\int_{\underline{\theta}}^{\theta}(1 - a'ß)d\theta dF$ and rearranging yields

$$\int_{\Theta} c(\theta)dF = \int_{\Theta} \frac{1-F}{f}(1 - a'ß)dF.$$

Substituting this expression into program 3 and ignoring the non–negativity and monotonicity constraints we obtain:

PROGRAM 4.

$$\max_{I,x,ß} \int_\Theta \left[\lambda(\theta - v - aß) + (1-\lambda)\frac{1-F}{f}(1 - a'ß)\right] dF - \lambda I \qquad \text{(P.4)}$$

As before, λ denotes the Langrange multiplier associated with the participation constraint (PC). First order conditions for x and $ß$ are still given by (3) and (4). Let $\hat{I}$ solve program 4 and I^* be the first best level of investment at which marginal investment equals expected marginal returns, i.e. I^* solves $1 = -\int_\Theta F_I \, d\theta$.

PROPOSITION 8 *If the incentive constraint binds at the optimal contract, investment is distorted away from its first best level. There may be over– or underinvestment.*
Assumptions 1, 3 and $U'' = 0$ *imply:*
(i) $\lambda = 1 \implies \hat{I} = I^*$;
(ii) $\lambda > 1 \implies \hat{I}$ *solves:*

$$1 = -\int_\Theta F_I \, d\theta + \frac{1-\lambda}{\lambda}\int_\Theta (1 - a'ß)\psi \, dF \qquad (7)$$

with

$$\psi \equiv \frac{\partial}{\partial I}\left(\frac{1-F}{f}\right) - \frac{F_I}{f}\frac{\partial}{\partial \theta}\left(\frac{1-F}{f}\right);$$

and (iii) $\frac{\partial}{\partial\theta}[f/F_I] \lesseqgtr 0 \iff \hat{I} \gtreqless I^*$.

According to (7) investment is optimal if marginal cost of investment, 1, equates its expected marginal return, (first term on the right hand side), adjusted by a term reflecting the subtle interaction of investment with ex post distortions. (By second order conditions and $1-\lambda < 0$, a positive ψ implies underinvestment). On the one hand, investment shifts the distribution towards good states for which distortions are low. This effect pushes towards more investment. It shows up in the second term of ψ, which is negative by $F_I < 0$ and $\frac{\partial}{\partial\theta}[(1-F)/f] < 0$. On the other hand, to the extend that investment increases the inverse hazard rate $\frac{\partial}{\partial I}[(1-F)/f] > 0$ it makes rent extraction and thereby costly distortion more necessary. This favours lower investment. The combined effect can be determined only in cases for which f/F_I is monotone in θ. For example, if investment shifts returns in a simple additive way, such that $F(\theta, I) = F(\theta + g(I))$ with $g' < 0$, then both effects would exactly cancel out, $\frac{\partial}{\partial\theta}[f/F_I] = 0$ and underinvestment would result only from marginal projects being cancelled. If the transformation takes a simple multiplicative form, such that $F(\theta, I) = F(\theta g(I))$ with $g > 0$, $g' < 0$ then $\frac{\partial}{\partial\theta}[f/F_I] > 0$ and we obtain underinvestment also on intramarginal projects.

5 Concluding Remarks

In this paper we analyze optimal financial contracts in a free–cash–flow framework. If the manager is risk neutral, pay–incentives become more distorted and managerial control becomes more restricted when performance is poor — in principal–agent jargon, there is 'distortion at the bottom'. The manager's consumption is convex in returns, hence optimal contracts introduce a preference for risk–taking. With proportional cost of appropriation, optimal contracts deviate from first best delegation of control by requiring investors to interfere upon poor performance but maintain managerial appropriation at its first best level. With convex appropriation cost, information rents may also be reduced by inducing inefficient appropriation. The more, the consumption in low states is obtained via appropriation, as opposed to straight pay, the less attractive it is to mimic these states when returns are high. This suggests that firms perform poorly because adverse conditions shatter pay–incentives. Convex cost also motivate the use of call–options on stock to compensate managers. With respect to investment, we characterize conditions under which agency-cost, by driving a wedge between the cost of internal and external funds, result in underinvestment. The results for risk–neutral managers lend themselves to interpretation in terms of financial contracts. However, by ruling out a fixed base wage, they conflict with an interpretation in terms of labour contracts. By introducing risk aversion, we obtain a fixed base wage, but relating the features to financial securities becomes more difficult.

In order to obtain a clear picture of the relation between payment, incentives and external interference we assumed that (i) financiers' activity is contractible and (ii) financiers and managers can commit to ex post inefficient outcomes. The former prevents us from analyzing how payouts and residual control rights are assigned among different groups of investors, which is central to the literature on corporate governance mentioned in the introduction. However, to the extend that financiers interfere with management activities they effectively become managers. Hence, the same considerations which govern payments and control assignments between management and financiers as a group should apply to the relations between different investors. In this paper investors' interference is necessary to achieve a steep increase of payout in bad states. If investors' activity is non–contractible, a steep increase is also necessary to provide them with the incentives to become active and take efficient decisions. This suggests that securities should have a seniority–structure with contingent control rights being assigned to those who are residual claimants.

Since final allocations are analyzed as pre–agreed outcomes, there is no role for contract renegotiation in this paper. Again, this prevents us from analyzing important features of the financial contract which may be instrumental in steering the outcome of contract renegotiation e.g. the number of financiers, Dewatripont & Tirole (1994), Bolton & Scharfstein (1996); the public/private nature of securities, Gertner & Scharfstein (1991); the term–structure and seniority of debt, Berglöf & von Thadden (1994); or the strategic use of collateral, Bester (1994). However, as Bester (1994) pointed out, contract renegotiation plays an ambiguous role if ad–hoc restrictions are imposed on the format of the initial contract — e.g. in order to keep the analysis of the renegotiation game tractable. On the one hand, renegotiation reduces welfare by constraining the ability to jointly commit. On the other hand, if some of the initial restriction are removed through renegotiation, the set of feasible allocations may be enlarged. In section 4 we dropped exogenous constraints on feasible allocations — apart from those resulting from incentive and wealth constraints. Hence, the parties would use whatever means they have at their disposal to achieve the allocations which we characterize as optimal.

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Technical Appendix

Proof of proposition 2.

With a proposition 2 contract, $\{s^*, A^*\}$, we can always decrease (increase) payments to the investor and control cost by reducing (increasing) D. Hence, it is sufficient to show that any other feasible contract $\{\hat{s}, \hat{A}\}$, yielding the same expected payments to manager and investors entails higher control cost.

Since the wealth constraint binds when investors are active, $\hat{s} \leq s^* \quad \forall x \in A^*$. If $\hat{s} \neq s^*$ for some x while $\int_X \hat{s} dF = \int_X s^* dF$ then there $\exists\, x_1 \in P^*$, such that $\hat{s}(x_1) > s^*(x_1)$. The wealth constraint implies $\hat{s}(D) \leq s^*(D) = D$. From $s^*(x_1) - D = \alpha(x_1 - D)$ follows that $\hat{s}(x_1) - (D + \epsilon) > \alpha(x_1 - (D + \epsilon))$ for some $\epsilon > 0$. To obey the incentive constraint all $x \leq D + \epsilon$ have to be elements of $\hat{A}$, implying that $\hat{A} \supset A^*$. If $\hat{s} = s^*$ then $\hat{A} \supseteq A^*$, because not imposing constraints for any $x \in A^*$ would violate the incentive constraint. Therefore, if $\{\hat{s}, \hat{A}\}$ is different from $\{s^*, A^*\}$ expected control cost are strictly higher. $\square$

Proof of Proposition 3.

Equation (1) follows from the first order conditions for the optimal choice of α and D. To obtain (ii), substitute the Lagrange-variable associated with (PC.2) in the sign–condition on the principal minor of the bordered Hessian ($|\bar{H}_2|$) using the first order condition. Rearranging yields:

$$\text{sign } |\bar{H}_2| = \text{sign } \frac{m''}{m'} - \frac{2}{1-\alpha} + \frac{m'}{vf}\left(f + (1-F)\frac{f'}{f}\right)$$

$\square$

Proof of proposition 4.

The contract of proposition 4 requires investors to pay a fixed compensation even when returns are very low. For $\underline{\theta}$ low enough this conflicts with limited liability as a feature of most financial securities. In order to analyze the issue in more detail we assume here that there is an exogeneous lower bound, $-L$ for payment s.

First, define the maximal pay–out for active financiers, $\phi : A \rightarrow \mathbb{R}$, which is incentive compatible given an arbitrary pay–out function in the case of investors remaining passive.

$$\phi(x; s, A) \equiv \min_{\tilde{x} \in P} [s(\tilde{x}) + \alpha |x - \tilde{x}|], \; x \in A$$

The incentive constraints require that $s(x) \leq \phi(x; s, A) \; \forall x \in A$.

The proof follows from a series of lemmas, each providing for a partial characterization of the solution for program 1.

LEMMA 1 *A contract* $\{s, A\}$ *solving program 1 entails* $s(x) < \phi(x; s, A), \quad \forall x \in A$

PROOF: Assume to the contrary that $\{\hat{s}, \hat{A}\}$ is optimal and that $A_1 \equiv \{x | x \in A, \text{ and } s(x) = \phi(x; s, A)\}$ is not empty. Now consider an alternative contract $\{\hat{s}, A^*\}$ with $A^* = \hat{A} \setminus A_1$. The contracts are alike, except that the latter does not impose controls on A_1 and saves control cost accordingly. $\{\hat{s}, A^*\}$ trivially fulfills the wealth and the liability constraints. It follows from the linearity of the incentive constraints (IC.1) that $\{\hat{s}, A^*\}$ is also incentive compatible. The new contract provides the same expected utility to the manager and makes the investor strictly better off by the control cost saved, $\int_{A_1} v dF$. This gain can be redistributed to the manager by a contract $\{s^*, A^*\}$ with $s^* = max\{\hat{s} - \eta, -L\}$ were η is chosen to provide the investor with the same reservation utility as under $\{\hat{s}, \hat{A}\}$. □

Lemma 2 and lemma 3 characterize the slope of the payment schedule for $x \in A$, respectively $x \in P$.

LEMMA 2 *When investors are active, the pay–out varies one to one with the return, except if the limited liability constraint binds:* $s(x) = \max\{x - c_1, -L\}, \quad \forall x \in A,$ *with* $c_1 \geq 0$

PROOF: Assume to the contrary that a different contract $\{\hat{s}, \hat{A}\}$ solves program 1. Replace it by $\{s^*, \hat{A}\}$ were

$$s^*(x) = \begin{cases} \hat{s}(x) & x \in \hat{P} \\ \min\{\max\{x - \eta, -L\}, \phi(x; \hat{s}, \hat{A})\}, & x \in \hat{A} \end{cases}$$

and $\eta \geq 0$ solves $\int_{\hat{A}}(s^* - \hat{s})dF = 0$. Note that $\hat{s} \leq s^*$ for $\eta = 0$ and $\hat{s} \geq s^*$ for $\eta = \bar{x} + L$. Since s^* is continuous in η, such a solution will always exist. Hence, by construction s^* yields the same expected payoff to the investor, but it keeps the managers consumption constant within the limits set by the incentive and the limited liability constraint. The distribution of $x - s^*$, therefore, dominates that of $x - \hat{s}$ according to second order stochastic dominance and will be preferred by the manager. If A_1 is not empty, we can invoke lemma 1 to obtain the desired contract $\{s^*, A^*\}$. □

LEMMA 3 *When investors remain passive the pay–out increases linearly with slope α, except if the limited liability constraint binds:*
$s(x) = \max\{\alpha x - c_2, -L\}, \quad \forall x \in P.$

PROOF: Assume to the contrary that a different contract $\{\hat{s}, \hat{A}\}$ solves program 1. Define $\check{s}(x) \equiv \max\{\alpha x - \eta, -L\}$, $\underline{\eta} \equiv -(1-\alpha)\inf[\hat{P}]$, $\bar{\eta} \equiv \alpha\bar{\theta} + L$ and consider a contract $\{s^*, \hat{A}\}$ with

$$s^*(x) = \begin{cases} \check{s}(x); & x \in \hat{P} \\ \min\{\hat{s}, \check{s}(x)\}; & x \in \hat{A} \end{cases}$$

were $\eta \in [\underline{\eta}, \bar{\eta}]$ solves $\int_X (s^* - \hat{s})dF = 0$. Since s^* is continuous in η, $\hat{s} \leq s^*$ for $\eta = \underline{\eta}$ and $\hat{s} \geq s^*$ for $\eta = \bar{\eta}$ such a solution will exist. Since $\check{s}(x) = \phi(x; s^*, \hat{A})$ all incentive, wealth and liability constraints are fulfilled by construction. If there exist an $x_1 \in \hat{P}$, such that $\hat{s}(x_1) < s^*(x_1)$ then $\hat{s}(x) < s^*(x) \, \forall x > x_1$, otherwise $\hat{s}$ would violate the incentive constraint. From this it is easily established that $x - \hat{s}$ is a mean preserving spread of $x - s^*$, and therefore inferior. □

Now we relate the two schedules:

LEMMA 4 *The parameters c_1, c_2 of lemma 2 and lemma 3 obey the following condition: $x - c_1 < \alpha x - c_2 \quad \forall \{x \in A | s(x) > -L\}$.*

PROOF: The claim follows from lemma 1. □

Lemmas 1 – 4 are about all, what can be derived without additional assumptions on probability distribution or risk preferences.

Lemma 5 *If the limited liability constraint does not bind, control takes place on a lower left hand interval:*
$\exists x_0$ *such that* $x \in A\ \forall x < x_0$*, and* $x \in P\ \forall x \geq x_0$

Proof: Assume to the contrary that a different contract $\{\hat{s}, \hat{A}\}$ obeying lemmas 2 and 3 is optimal. Define: $\hat{s}_N(x) = \alpha x + \hat{c}_2$, $\hat{s}_A(x) = x - \hat{c}_1$, $x_0 = \inf[\hat{A}]$, $x_1 = \sup[\hat{A}]$, $\eta_0 = \hat{s}_N(x_0) - \hat{s}_A(x_0)$, $\eta_1 = \hat{s}_N(x_1) - \hat{s}_A(x_1)$. Lemmas 2 and 3 imply $\eta_0 > \eta_1$ if the liability constraint is not binding at x_0. Then we can choose $A_1 \subseteq \hat{A}$ and $P_1 \subseteq \hat{P}$ such that (i) $x_N < x_A$, $\forall\ x_N \in P_1$, $x_A \in A_1$, (ii) $\int_{P_1} dF = \int_{A_1} dF$ and (iii) $\hat{s}_N(x_N) - \eta_1 > L$, $\forall\ x_N \in P_1$. Consider another contract $\{s^*, A^*\}$ with $A^* = P_1 \cup \hat{A} \setminus A_1$ and

$$s^*(x) = \begin{cases} \hat{s}(x); & x \in \hat{P} \setminus P_1 \\ \hat{s}_N - \eta; & x \in P_1 \\ \hat{s}_N; & x \in A_1 \\ \hat{s}(x); & x \in \hat{A} \setminus A_1 \end{cases}$$

$\eta \in [0, \eta_1]$ is chosen so that $\int_{P_1 \cup A_1} (\hat{s} - s^*) dF = 0$. Since s^* is continuously decreasing in η and $s^* > \hat{s}$ for $\eta = 0$ and $\int_{P_1 \cup A_1} (\hat{s} - s^*) dF > 0$ for $\eta = \eta_1$ such a solution will exist. The new contract provides the investor with the same expected pay–out and does not entail higher control cost. The manager's consumption is increased (decreased) on P_1 (A_1) where it was low (high) but not above (below) the original level at A_1 (P_1). The distribution of $x - \hat{s}$ is a mean preserving spread of the corresponding distribution of $x - s^*$, hence $\{s^*, A^*\}$ is superior. □

Lemma 6 *If the limited liability constraint does not bind, the optimal payment schedule (the manager's consumption) has an upward (downward) jump where investors turn passive, except if the wealth–constraint binds at that point:*
Define $x_0 \equiv \inf[P]$. $\alpha x_0 - c_2 < x_0 \Longrightarrow x_0 - c_1 < \alpha x_0 - c_2$.

Proof: Note that since s is continuous on A, $x_0 - c_1 > \alpha x_0 - c_2$ would violate lemma 4. Using lemmas 2, 3 and 5 we can rewrite the optimization problem as on in three variables $\{c_1, c_2, x_0\}$. Inspection of first order condition reveals that $x_0 - c_1 = \alpha x_0 - c_2$ cannot be optimal. □

Proposition 4 summarizes lemmas 2, 3, 5, 6 with obvious substitutions as: $w_1 \equiv c_1$, $D \equiv s(\inf[N])$ and $w_2 \equiv \inf[N] - D$ □

Preparing Propositions 5, 6 and 7.

Define

$$\begin{aligned}\mathcal{L} &= U(c)f + \lambda[\theta - v(ß) - c - a(\theta - x)ß - I]f \\ &\quad + \psi[1 - a'(\theta - x)ß + g] + \mu_1[\theta - x] + \mu_2[\theta - x]g\end{aligned}$$

Ignoring the monotonicity constraint (M) for the time being and considering $ß$ as exogeneously fixed, the solution to program 3 is characterized by the following (additional) conditions:

$$0 \geq \lambda f a' ß + \psi a'' ß - \mu_1 - \mu_2 g, \qquad 0 = [\lambda f a' ß + \psi a'' ß - \mu_1 - \mu_2 g] \cdot x \tag{8}$$

$$0 \leq \psi + \mu_2(\theta - x), \qquad 0 = [\psi + \mu_2(\theta - x)]g \tag{9}$$

$$0 \leq \mu_1, \qquad 0 = (\theta - x)\mu_1 \tag{10}$$

the costate-equation

$$\psi' = (\lambda - U')f \tag{11}$$

and the transversality conditions:

$$0 \geq \psi(\underline{\theta}), \qquad 0 = [c(\underline{\theta}) - (\underline{\theta} - a(\underline{\theta} - x(\underline{\theta}))ß(\underline{\theta}))] \cdot \psi(\underline{\theta}) \tag{12}$$

$$0 = \psi(\bar{\theta}) \tag{13}$$

Where λ, μ_1, μ_2 and ψ denote multipliers associated with (PC.3), (NF.3), (ICb.3) and the costate–variable, respectively.

To ease notation define:

$$\begin{aligned}EU' &\equiv E[U'] = \int_{\underline{\theta}}^{\bar{\theta}} U'(c(z))dF(z) \\ EU'_-(\theta) &\equiv E[U'(c(z))|z \leq \theta] = \int_{\underline{\theta}}^{\theta} U'(c(z))dF(z)/F(\theta) \\ EU'_+(\theta) &\equiv E[U'(c(z))|z > \theta] = \int_{\theta}^{\bar{\theta}} U'(c(z))dF(z)/(1 - F(\theta))\end{aligned}$$

Integrate (11)

$$\begin{aligned}\psi &= \lambda F - \int_{\underline{\theta}}^{\theta} U'(c(z))dF(z) + k \\ &= F(\lambda - EU'_{-}) + k \qquad (14)\end{aligned}$$

Evaluation at $\underline{\theta}$ and $\bar{\theta}$ yields

$$\psi(\underline{\theta}) = k, \qquad 0 = \lambda - EU' + k, \qquad (15)$$

Proof of Proposition 5.

With risk–neutrality $EU' = EU'_{-} = U'$ and we have to assume that the wealth constraint is strictly binding at $\underline{\theta}$, implying $\psi(\underline{\theta}) < 0$, to obtain a specific solution.[20] From (12) we have $k = U' - \lambda < 0$. Substituting k in (14) yields

$$\psi(\theta) = (1 - F(\theta))(U' - \lambda) < 0, \quad \forall \theta < \bar{\theta} \qquad (16)$$

For $x = \theta$, (9) implies (for $x < \theta$ the same follows from (ICb.3))

$$g(\theta) = 0. \qquad (17)$$

Substitute ψ in (8) to obtain for $x > 0$:

$$0 = a' ßf\lambda - a'' ß(1 - F)(\lambda - U') - \mu_1 \qquad (18)$$

Recall that $ß = 1$. Complementary slackness (10) requires $\mu_1 = 0$ for $x < \theta$ which leaves us with (3).

Slope of x. The solution to the relaxed program 3 is only valid if $x' > 0$. Here, we derive the stronger result that $x' > 1$ for $x < \theta$. Define $L_x \equiv a'f - a''(1 - F)(1 - U'/\lambda)$. Since $L_x \equiv 0$ for $x < \theta$, $x' = -L_{x\theta}/L_{xx}$. By assumption 3

$$L_{xx} = a'''(1 - F)(1 - U'/\lambda) - a''f < 0$$

[20] For $I < I^0$ only the participation constraint binds and $\lambda = U'$. In this case $\psi \equiv 0$ and it follows from (18) that $\mu_1 > 0 \Longrightarrow x = \theta \quad \forall \theta$. However, g cannot be determined from (9).

as required by the second order conditions. Since, $L_{x\theta} = a'f' + fa''(1 - U'/\lambda) - L_{xx}$

$$x' = 1 + \frac{a'f' + fa''(1 - U'/\lambda)}{-L_{xx}}$$

To prove that $x' > 1$ we show that

$$\frac{f'}{f} + \frac{a''}{a'}(1 - U'/\lambda) > 0$$

$L_x = 0$ implies

$$\frac{a''}{a'}(1 - U'/\lambda) = \frac{f}{1 - F}$$

Substitution yields

$$\frac{f'}{f} + \frac{f}{1 - F} > 0$$

which is an implication of assumption 1.

Features of s. The claims on the slope of $s(\theta)$ follow from differentiating the identity (2) and substitution with (ICa). □

Proof of corollary 1.

Since $x'(\theta) > 1$ for $\theta > \check{\theta}$ it can be inverted to obtain $\theta(x)$, $x \in (0, \bar{\theta}]$ with $\theta' = 1/x' < 1$. Substitution in the identity (2) and differentiation yields

$$s' = \theta' \cdot (1 - c' - a') + a'$$

substitute with (ICa) to obtain

$$s' = a' > 0; \qquad s'' = a'' \cdot (\theta' - 1) < 0$$

□

Proof of Proposition 6.

For an interior solution of ß being optimal, we have to add

$$0 = -\lambda f(v' + a) - \psi a' \tag{19}$$

to the conditions stated above, which in combination with (16) implies equation (4). First order conditions are sufficient, because:

$$\begin{aligned} \mathcal{L}_{ßß} &= -\lambda f v'' < 0 \\ \mathcal{L}_{ßx} &= \lambda f a' + \psi a'' = 0 \quad \text{by} \quad \mathcal{L}_x = 0 \end{aligned}$$

The latter implies that the statements referring to x hold true by the same arguments as in proposition 5. $ß' < 0$ follows from:

$$\mathcal{L}_{ß\theta} = -\lambda f'(v' + a) - \psi' a' - \mathcal{L}_{ßx}$$

rearranging and substitution with the help of (19) yields:

$$\mathcal{L}_{ß\theta} = -\lambda(v' + a)\left(f' + \frac{f^2}{1-F}\right) < 0$$

which is negative by assumption 1. □

Proof of Proposition 7.

With risk aversion it is assumed that the wealth constraint is not binding at $\underline{\theta}$, implying $c(\underline{\theta}) > 0$ and $\psi(\underline{\theta}) = 0$. In this case the sole purpose of costly appropriation is to improve insurance. As argued in the text, a complete solution would require the incorporation of the monotonicity constraint into the program. Fortunately, the solution is characterized by the relaxed program on the set on which the constraint is not binding. For the partial characterization of proposition 7 it can, therefore, be ignored.

From (15) and (12) we have $k = EU' - \lambda = 0$. Substituting λ in (14) yields

$$\psi(\theta) = -F(\theta)(EU'_{-} - EU') < 0, \quad \forall\theta \in (\underline{\theta}, \bar{\theta}) \tag{20}$$

Since $g = 0$ by (9) and (ICb.3) we can rewrite (8) for $x > 0$ as:

$$0 = EU' a' ßf - F(EU'_{-} - EU')a'' ß - \mu_1 \tag{21}$$

Complementary slackness requires $\mu_1 = 0$ for $x < \theta$. Using $EU' = FEU'_- + (1-F)EU'_+$, equation (21) can be rewritten as (6).

To show that distortion takes place in the middle define $L_x \equiv EU'a'f - F(EU'_- - EU')a''$. For $\theta \in (\underline{\theta}, \bar{\theta})$, $F(EU'_- - EU') > 0$ hence $a''(0)/a'(0)$ can be made large enough, given the degree of risk aversion, to make L_x negative when evaluated at $x = \theta$. Hence $\Theta_m \neq \emptyset$. From $\lim_{\theta \to \bar{\theta}} EU'_- - EU' = 0$ and $F(\underline{\theta}) = 0$ follows $L_x(\underline{\theta}), L_x(\bar{\theta}) > 0$ implying $\underline{\theta}, \bar{\theta} \notin \Theta_m$.

Slope of x. Finally, it is shown that sign x' is equivalent to expression (5) whenever the monotonicity constraint is slack. x' is given by $x' = -L_{x\theta}/L_{xx} > 0$. By assumption 3

$$L_{xx} = -a'''F(EU'_- - EU') - a''fEU' < 0$$

as required by the second order conditions.

$$L_{x\theta} = EU'(a''f - a'f) - (EU'_- - EU')(a''f + a'''F) - a''FU'$$

From $L_x = 0$ follows $(EU'_- - EU') = EU'a'f/a''F$, substitution and division by $EU'a'f > 0$ yields:

$$\text{sign } L_{x\theta} = \text{sign } \left(\frac{a''}{a'} - \frac{a'''}{a'}\right) + \left(\frac{f'}{f} - \frac{f}{F}\right) - \frac{F}{f}\frac{a''}{a'}\frac{U'}{EU'}$$

The right hand side is equivalent to expression (5). □

Proof of proposition 8.

To ease notation let B denote the square bracketed integrant in program 4. The first order condition for the optimal choice of I is:

$$0 = Bf\frac{d\theta}{dI}\bigg|_{\underline{\theta}}^{\bar{\theta}} + \int_\Theta Bf_I\, d\theta + \int_\Theta (1-\lambda)(1-a'\beta)\frac{\partial}{\partial I}\frac{1-F}{f} f\, d\theta - \lambda \qquad (22)$$

Partial integration of the second term yields:

$$\begin{aligned}\int_\Theta Bf_I\, d\theta &= BF_I|_{\underline{\theta}}^{\bar{\theta}} - \int_\Theta \frac{dB}{d\theta}F_I\, d\theta \\ &= BF_I|_{\underline{\theta}}^{\bar{\theta}} - \int_\Theta \left(\frac{\partial B}{\partial \theta} + \frac{\partial B}{\partial x}x' + \frac{\partial B}{\partial \beta}\beta'\right) F_I\, d\theta \qquad (23)\end{aligned}$$

Since $F(\underline{\theta}(I), I) = 0$ respectively $F(\bar{\theta}(I), I) = 1$, $\forall I$ we can substitute in (22) with $d\underline{\theta}/dI = -F_I(\underline{\theta}, I)/f(\underline{\theta}, I)$ and $d\bar{\theta}/dI = -dF_I(\bar{\theta}, I)/f(\bar{\theta}, I)$. Hence the first term in 22 and in 23 cancel out. If $\lambda = 1$ first order conditions imply $\text{ß} = 1$ and $x = \theta$ everywhere. It follows that $\text{ß} = 0$, $x' = 1$ and $\partial B/\partial\theta = 1 - \partial B/\partial x$. Therefore, $dB/d\theta = 1$ and (22) reduces to $1 = -\int_\Theta F_I \, d\theta$, which proves (i).

For $\lambda > 1$, first order conditions require $\partial B/\partial x = \partial B/\partial \text{ß} = 0$ implying:

$$\begin{aligned} \frac{dB}{d\theta} &= \lambda(1 - a'\text{ß}) - (1-\lambda)\frac{1-F}{f}a''\text{ß} \\ &\quad + (1-\lambda)(1-a'\text{ß})\frac{\partial}{\partial\theta}\frac{1-F}{f} \end{aligned}$$

By first order condition for x, (3), the term in the first row simplifies to λ. Hence (22) can be rewritten as

$$\begin{aligned} 0 &= -\lambda \int_\Theta F_I \, d\theta \\ &\quad - \int_\Theta (1-\lambda)(1-a'\text{ß})\frac{\partial}{\partial\theta}\frac{1-F}{f} F_I \, d\theta \\ &\quad + \int_\Theta (1-\lambda)(1-a'\text{ß})\frac{\partial}{\partial I}\frac{1-F}{f} \, dF - \lambda \end{aligned}$$

Use $d\theta = dF/f$ and rearrange to obtain (7). Since $1-\lambda < 0$ and $1 - a'\text{ß} > 0$ it remains to be shown that sign $\psi =$ sign $\frac{\partial}{\partial\theta}[f/F_I]$ to prove (iii). Rewrite ψ as:

$$\psi = \frac{F_I^2}{f^2}\frac{1-F}{f}\frac{\partial}{\partial\theta}\left(\frac{f}{F_I}\right)$$

$\square$