Investment and Risk–taking in Procurement and Regulation

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Abstract

This note analyses investment and risk–taking in a simple agency model of public regulation/procurement borrowed from Laffont & Tirole (1993). We show that the principal will overinvest or underinvest depending on whether investment is marginally more productive in bad or in good states. Due to the incentive problem investment risk will be avoided even though all parties are risk–neutral. Apparently, this 'excessive' risk–aversion extends to a large class of agency problems in organization theory and finance.

JEL: L51, D82, H57

Introduction

Following Laffont & Tirole (1993) we consider a principal, also referred to as 'government', hiring an agent, the 'firm', to manage a public project. Both parties are risk–neutral. The principal makes an investment up front and uses an incentive scheme in order to elicit non–contractible effort for cost reduction. How does investment interact with the incentive problem? What are the conditions for optimal investment being distorted away from its first best level and how are these conditions to be interpreted? As shown in Laffont & Tirole (1993) the incentive problem may lead both to under and overinvestment. This note provides for an intuitive condition determining which bias prevails and relates it to the risk–shifting properties of investment. In spite of the assumed risk–neutrality optimal investment deviates from first best investment as to reduce project risk. Hence incentive problems offer an alternative explanation for risk–averse behaviour which does not built on exogenous assumptions about preferences.

In order to save space, we limit the formal analysis to a particular model of regulation, analyzed in more detail in the well known textbook of Laffont & Tirole (1993). After outlining the model in the first two sections, we show that the incentive problem generates risk–aversion with respect to investment. In the concluding section, we argue that this result, can be derived from some fairly basic features of the agency problem. Therefore, it is also relevant for organization theory and finance.

The Principal–Agent–Problem

The cost of the project depend on investment I, the agent's effort $e \in [0, \infty]$ and an efficiency parameter $\beta \in [\underline{\beta}(I), \bar{\beta}(I)]$, according to $C = I + \beta - e$. The efficiency parameter β is a random variable distributed with $F(\beta|I)$ and density $f(\beta|I)$. Investment I improves the distribution of β in the sense that high realization of β become less likely. Hence, by investing up–front the principal can improve efficiency ex–post.[1] While the agent can observe the realization of β before deciding about e the principal knows only its stochastic

[1] Technically we require that F is log–concave (implying that $d(f/F)/d\beta < 0$), $F_I > 0$ (first order stochastic dominance), $d(f/F)/dI \leq 0$ (hazard rate dominance) and a decreasing rate of return $F_{II} < 0$. Note, that the usual criteria for first order stochastic dominance and hazard rate dominance have been adapted, since β is a cost.

properties. The agent's disutility (in monetary units) of expending effort to reduce cost is an increasing and convex function $\psi(e)$.[2] The principal can neither observe effort nor disutility but he knows the agent's preferences. As an accounting convention cost C is reimbursed. In addition the firm receives a net–transfer of t in compensation for its effort. The firm's payoff U in state β is thus given as: $U(\beta) = t(\beta) - \psi(e(\beta))$.

The agent cannot be prevented from abandoning the project after learning β. Accordingly, the firm will honour the contract only if the pay–off exceeds its reservation utility which is normalized to zero yielding the ex–post participation constraint:

$$U(\beta) \geq 0; \qquad \forall \beta. \tag{1}$$

Since both e and β are unobservable, effort cannot be inferred from observed cost, hence the contract has to be incentive compatible. By the revelation principle we can focus on direct incentive compatible mechanisms, stipulating effort $e(\beta)$ and rent $U(\beta)$ as functions of the unobservable parameter in such a way that it is in the best interest of the firm to reveal β truthfully. Define the firm's payoff as a function of the true state β and its message $\tilde{\beta}$ (possibly false): $\varphi(\beta, \tilde{\beta}) = t(\tilde{\beta}) - \psi(\beta - C(\tilde{\beta}))$. Incentive compatibility requires

$$\varphi(\beta, \beta) \geq \varphi(\beta, \tilde{\beta}), \quad \forall \quad \beta, \tilde{\beta}. \tag{2}$$

i.e. the firm's rent when telling the truth, $\tilde{\beta} = \beta$ (the left side) must not be smaller than when falsely claiming $\tilde{\beta} \neq \beta$ (the right side).

The project has value S for consumers. In order to finance cost and transfer distortionary taxes have to be used which inflict a loss of \$ $1 + \lambda$ for every \$ raised. The parameter $\lambda > 0$ indicates the deadweight loss of taxation. Hence, the net surplus of consumers/taxpayers is $S - (1+\lambda)(t + \beta - e - I)$. Adding the firm's utility and substituting for t yields ex post social welfare as

$$w(\beta) = S - (1+\lambda)(I + \beta - e + \psi(e)) - \lambda U$$

The government's aim is to select I, e and U in order to maximize ex–ante expected welfare

$$\max_{I,e,U} W = \int_{\underline{\beta}}^{\bar{\beta}} w(\beta) dF(\beta|I), \qquad \text{s.t. (1) and (2).} \tag{3}$$

[2]Formally we require $\psi' > 0, \psi'' > 0$, $\psi(0) = 0$, $\lim_{e \to \beta} = \infty$ and $\psi''' \geq 0$.

The combination of non–contractibility of effort and limited liability creates a trade–off between rent–extraction and incentives for cost reduction. Powerful incentives could be obtained with a 'fixed–price' contract, i.e. by making the agent the residual claimant. If payment does not depend on realized cost, the firm would choose the first best level of effort in all states. However, such a contract would leave the firm with excessive rents, because the fixed payment would have to be generous enough to ensure participation even in the worst possible state. The other extreme would be a 'cost–plus' contract which, by raising payment one to one with observed cost, destroys any incentives for cost reduction. The firm would never exert any effort but could also be denied any rent by setting the 'plus' equal to zero.

The Optimal Contract

In order to derive the optimal contract we make use of the incentive constraint and the limited liability constraint to substitute U and restate problem (3) more conveniently as:[3]

$$\begin{aligned}\max_{I,e(\beta)} W &= S-(1+\lambda)I \\ &\quad -\int_{\underline{\beta}}^{\bar{\beta}}\left[(1+\lambda)(\beta-e+\psi(e))+\lambda\frac{F(\beta|I)}{f(\beta|I)}\psi'(e)\right]dF(\beta|I)\end{aligned} \tag{4}$$

First order conditions for the optimal choice e and I imply:[4]

$$(1+\lambda)[1-\psi'(e(\beta))]f(\beta|I)=\lambda F(\beta|I)\psi''(e(\beta)), \quad \forall\beta \tag{5}$$

$$1=\int_{\underline{\beta}}^{\bar{\beta}} F_I d\beta+\frac{\lambda}{1+\lambda}\int_{\underline{\beta}}^{\bar{\beta}}\psi'(e(\beta))\cdot\gamma\cdot dF \tag{6}$$

with

$$\gamma\equiv\frac{F_I}{f}\cdot\frac{\partial}{\partial\beta}\left(\frac{F}{f}\right)-\frac{\partial}{\partial I}\left(\frac{F}{f}\right)$$

[3] For a brief summary of the main steps see the appendix. For a detailed exposition see Laffont & Tirole (1993).

[4] Given our assumptions these conditions are also sufficient. They correspond to equations (1.44) and (1.71) in Laffont & Tirole (1993).

Equation (5) reflects the well understood trade off between rent extraction and effort–incentives. The left hand side, gives the expected returns from raising effort marginally in state $\beta > \underline{\beta}$. This requires, however, improving incentives by granting the firm a larger share of cost–saving. The firm's rent must be increased by ψ'' in all states better than β, and the right hand side of (5) reflects the social cost doing so. Since these cost are zero at $\underline{\beta}$, effort is first best in the best state, $1 = \psi'(e(\underline{\beta}))$. This is a rather common feature of optimal incentive contracts, sometimes referred to as 'no distortion at the top'.[5] By differentiating (5) it can be established that $e'(\beta) < 0$. Hence, incentives get weaker and optimal effort declines as cost increases ('distortion at the bottom').

Equation (6) characterizes optimal investment I^*. In the absence of the agency problem, or if rents were costless, first best investment I_o would solve $1 = \int_{\underline{\beta}}^{\bar{\beta}} F_I d\beta$. Equation (6) reveals that the marginal return to investment has to be adjusted for the impact on expected agency cost. By second order conditions we obtain overinvestment ($I^* > I_o$) if the additional term is positive and underinvestment if it is negative. Since ψ' is non–negative, the direction of the distortion can be established if we can sign γ, which consists of two terms with opposite signs. On the one hand investment reduces the probability of β being high for which the efficiency loss is large. This effect pushes towards overinvestment (the positive first term). On the other hand it makes rent extraction and therefore costly distortions more necessary, which pushes towards underinvestment (the negative second term).

Risk–shifting and Investment

So far we have summarized the results of Laffont & Tirole (1993). In order to shed additional light on the somewhat inconclusive result regarding optimal investment we establish a sufficient condition which allows us to sign γ, respectively the investment bias.

PROPOSITION 1
There is overinvestment (underinvestment) if $d(F_I/f)/d\beta > 0 \quad (< 0)$.

[5] This mnemonic stems from models in which the best state corresponds to the 'top' of the distribution of the random variable. Here we have the reverse case.

PROOF: To prove the claim we show that sign $d(F_I/f)/d\beta = \text{sign}\ \gamma$. Use

$$\frac{\partial}{\partial\beta}\left(\frac{f}{F_I}\right) = \frac{f' F_I - f_I f}{F_I^2}$$

to rewrite γ as

$$\gamma = -\frac{F}{f} \cdot \left(\frac{F_I}{f}\right)^2 \cdot \frac{\partial}{\partial\beta}\left(\frac{f}{F_I}\right), \tag{7}$$

hence γ is positive (negative) if

$$\frac{\partial}{\partial\beta}\left(\frac{F_I}{f}\right) > 0 \quad (< 0), \quad \forall\beta.$$

□

The implicit dependence of β on I given by $F(\beta|I)$ is all we need for the formal analysis. In order to ease the interpretation, however, we introduce a new random variable 'state of nature' θ and consider the explicit function $\beta(I,\theta)$ with $\beta_I < 0$, and $\beta_\theta > 0$ by convention. Then F_I/f is increasing if marginal cost saving is larger in states for which cost are high $(-\beta_{I\theta} > 0)$ and it is decreasing if cost reductions are larger in those states when cost are already low $(-\beta_{I\theta} < 0)$. In the former case marginal returns are larger in bad states whereas in the latter case marginal productivity is larger in good states. Hence proposition 1 claims, that it is optimal to overinvest (underinvest) if marginal investment is more productive in bad (good) states.

Next, we relate the slope of F_I/f to the risk–shifting properties of investment. Let $k \equiv I + \beta$ denote ex–post cost, ignoring efforts to reduce cost, and let $G(k|I) = F(k - I|I)$ be the corresponding probability distribution. By accounting for both, the efficiency gains and the investment cost, expected pre–effort cost k measures the net return to investment.

PROPOSITION 2
If $d(F_I/f)/d\beta > 0 \quad (< 0)$ then a marginal increase of I evaluated at I_o reduces (increases) the risk of pre–effort cost k.

PROOF: Expressed in terms of G first best investment I_o requires:

$$0 = \int_{\underline{k}}^{\bar{k}} G_I(k|I_o)dk = \int_{\underline{k}-I}^{\bar{k}-I} (-f + F_I)\, d\beta = \int_{\underline{k}-I}^{\bar{k}-I} \left(\frac{F_I}{f} - 1\right) f d\beta. \tag{8}$$

From the last expression it is obvious, that for I_o and $\hat{k} < \bar{k}$

$$\frac{\partial}{\partial\beta}\left(\frac{F_I}{f}\right) > 0 \quad \Longrightarrow \int_{\underline{k}}^{\hat{k}} G_I(k|I_o)dk < 0 \qquad \forall \hat{k} < \bar{k}. \tag{9}$$

From (9) follows that for $d(F_I/f)/d\beta > 0$, and a sufficiently small $h > 0$

$$\int_{\underline{k}}^{\hat{k}} [G(k|I_o) - G(k|I_o + h)]dk > 0 \qquad \forall \hat{k} < \bar{k}$$

Hence, $G(k|I_o+h)$ is less risky then $G(k|I_o)$ according to second order stochastic dominance. Together with (8) we obtain the special case of a mean preserving compression of the distribution G. The proof for the risk increasing case is analog. □

Combining the two propositions we find that overinvestment occurs when investment decreases risk whereas underinvestment takes place when investment increases risk.[6] In both cases risk is reduced by deviating from the first best investment. Although all parties are risk neutral it is optimal to avoid investment risk in order to alleviate the incentive problem. Thus risk averse behaviour might be a result of incentive problems rather than preferences.

Concluding Remarks

The last section provided an alternative interpretation for the distortion of investment in Laffont & Tirole's (1993) model of public regulation and procurement. While some features of the model are akin to cost–saving under regulation, the basic trade–off between effort–incentives and rent–extraction is common. For example, the top–management of a private firm faces a similar task of providing incentives for cost–cutting to its divisional managers and financial contracts must reward financiers while maintaining incentives for the firm. It would, therefore, be useful to know whether the same bias of investment can be derived in other agency settings. A look at the related literature provides a tentative 'yes' to this question and helps to identify the assumptions which drive the result.

Townsend (1979) analyses optimal financial contracts in a setting in which a risk neutral firm with limited liability can costlessly appropriate the returns

[6] Unfortunately, if F_I/f is not monotone, we can neither sign the investment bias nor establish the risk–shifting properties of investment.

of the project. Financiers can prevent stealing by verifying the true state at some cost. As in our model of regulation, it is important to extract rents from the firm in order to obtain external funding in the first place. In bad states repayment is constrained by low returns and raising payouts as returns get better would induce the firm to steal and strategically claiming default. While it is not optimal to sacrifice on incentives in Townsend's model, the optimal contract features distortions in bad states, when financiers incure verification cost, and no distortions in good states, when investors remain passive. Gale & Hellwig (1986) address investment in this model and obtain underinvestment for a specific case in which investment is marginally more productive in good states, hence of the risk–increasing type. It is straightforeward to derive overinvestment by modifying their assumption on investment technology. Hubert (1999) looks at investment in various financial contracting models including Diamond's (1984) model of debt contracts which differs from Townsend's approach by allowing lenders to punish borrowers in case of default. He also obtains deviations from first best investment which help to reduce project risk.

While these models consider very different forms of ex–post inefficiencies (too little effort, costly verification, punishments) and obtain different optimal payments schedules, they do share a common basic structure, namely:

1. The agent is risk neutral but lacks wealth, or is protected by limited liability, or can abandon the project — the ex–post–participation constraint.

2. Knowing the state of nature, he makes a non–contractible choice to withhold effort, steal returns etc. — yielding the incentive constraint.

3. Leaving rents with the agent is costly for the principal.

4. The two sides can commit to ex–post inefficient outcomes. Cost and utility functions are such that optimal contracts feature 'distortions at the bottom' and 'no distortions at the top'.

Apparently, 'risk–aversion with respect to investment' is a feature of all models sharing these essentials. If this impression is correct, then incentive problems would offer for an endogeneous explanation of risk–aversion in a wide range of circumstances.

Technical Appendix

Rewriting the principal's problem.

As in Laffont & Tirole (1993) we replace the incentive constraint (2) by the first order condition for being truthful, $\varphi_2(\beta, \beta) = 0$ which implies

$$U'(\beta) = -\psi'(e(\beta)), \tag{10}$$

and the monotonicity constraint

$$e'(\beta) \leq 1 \tag{11}$$

which ensures fulfillment of the second order condition. Since (11) is fulfilled at the solution of the unconstrained program it will be ignored henceforth. Integration of (10) yields:

$$U(\beta) = \int_{\beta}^{\bar{\beta}} \psi'(e(\tilde{\beta}))d\tilde{\beta} + U(\bar{\beta}).$$

Because of $U'(\beta) < 0$ the wealth constraint (1) will bind only at $\bar{\beta}$ and since rents are socially costly, it is optimal to set $U(\bar{\beta}) = 0$. Integrating by parts and rearranging yields the following expression for the expected rent of the firm which incorporates the first order condition for truth–telling and the wealth constraint:

$$\int_{\underline{\beta}}^{\bar{\beta}} U(\beta)dF(\beta|I) = \int_{\underline{\beta}}^{\bar{\beta}} \frac{F(\beta|I)}{f(\beta|I)}\psi'(e(\beta))dF(\beta|I) \tag{12}$$

Substituting this into (3) we obtain (4).

Deriving the first order condition for investment

To ease notation let H denote the square bracketed integrant in program 3. The first order condition for the optimal choice of I is:

$$(1+\lambda) = -\left. Hf\frac{d\beta}{dI}\right|_{\underline{\beta}}^{\bar{\beta}} - \int_{\underline{\beta}}^{\bar{\beta}} Hf_I\,d\beta - \int_{\underline{\beta}}^{\bar{\beta}} \lambda\psi'\frac{\partial}{\partial I}\left(\frac{F}{f}\right)f\,d\beta \tag{13}$$

Partial integration of the second term yields:

$$\int_{\underline{\beta}}^{\bar{\beta}} H f_I \, d\beta = H F_I|_{\underline{\beta}}^{\bar{\beta}} - \int_{\underline{\beta}}^{\bar{\beta}} \frac{dH}{d\beta} F_I \, d\beta \tag{14}$$

Since $F(\underline{\beta}(I), I) = 0$ respectively $F(\bar{\beta}(I), I) = 1$, $\forall I$ we can substitute in (13) with $d\underline{\beta}/dI = -F_I(\underline{\beta}, I)/f(\underline{\beta}, I)$ and $d\bar{\beta}/dI = -F_I(\bar{\beta}, I)/f(\bar{\beta}, I)$, so that the first terms in 13 and in 14 cancel out.

Optimality of e requires $\partial H/\partial e = 0$, hence

$$\frac{dH}{d\beta} = \frac{\partial H}{\partial \beta} = (1+\lambda) + \lambda \psi'(e) \frac{\partial}{\partial \beta}\left(\frac{F}{f}\right),$$

which allows us to rewrite (13) as

$$\begin{aligned}(1+\lambda) \quad = \quad & (1+\lambda)\int_{\underline{\beta}}^{\bar{\beta}} F_I \, d\beta + \lambda \int_{\underline{\beta}}^{\bar{\beta}} \psi'(e) \frac{\partial}{\partial \beta}\left(\frac{F}{f}\right) F_I \, d\beta \\ & - \lambda \int_{\underline{\beta}}^{\bar{\beta}} \psi'(e) \frac{\partial}{\partial I}\left(\frac{F}{f}\right) f \, d\beta\end{aligned}$$

Substitution of variable using $d\beta = dF/f$ and rearranging yields equation (6).

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