Net Worth, Risk Taking and Investment

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[0] An earlier version of this paper circled under the title 'What is Wrong with the 'Balance Sheet Channel'?'

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Abstract

This paper analyzes how agency problems in financial contracting determine risk–taking and investment. In perfect capital markets a risk–neutral firm would invest until the expected marginal return equals the interest rate. However, as firms with little net–worth face agency cost in financial contracting the shadow cost of external funding are increased. It is often argued that this wedge forces firms to underinvest (the 'balance–sheet–effect'). However, this claim is premature. Firms with low net worth, hence high cost of external financing, might even overinvest. We establish a robust though more subtile balance sheet effect. Agency problems in financial contracting generate risk avoidance at the firm level even when all actors are risk neutral. This may explain why during major economic downturns firms and banks shun risks that could be accepted in better times.

Keywords: financial contracts, agency cost, risk, investment, balance sheet effect, business cycle
JEL E44, G3, D82

1 Introduction

According to Irving Fisher's well known separation–theorem in perfect capital markets investment would be independent of the investor's time–preferences and wealth. When confronted with the rapid decline in economic activity during the Great Depression however, Fisher (1933) went beyond 'separation', and argued that firms were selling assets and cutting back on investment in an attempt to lower their debt–burden. As many firms did so simultaneously, distress sales met weak demand sending prices to the bottom. The resulting deflation led to a further decline of firms' net worth by increasing the real value of debt, initiating thereby another round in the downward spiral. To cut through this vicious cycle, dubbed 'debt–deflation', he recommended a 'reflation of the dollar' to improve firms' wealth and revive investment. Fisher also observed a distortion of interest rates in particular that, 'money interest on safe loans falls but money interest on unsafe loans rises' (p. 343). But he did not relate the increase of the risk–premium to the decline of net–worth.[1]

Clearly, 'debt–deflation' requires some sort of capital market failure, otherwise firms would not care about indebtedness in the first place. For half a century, however, mainstream economic theory focussed on the implications of perfect capital markets — and Fisher became renowned for 'separation' rather than 'debt–deflation'. By the early eighties, advances in the analysis of imperfect information and incentives sparked new interest in the potential of financial markets to play an active role in the business cycle (Bernanke (1983), Blinder & Stiglitz (1983), Mankiw (1986), Mishkin (1991)). In particular agency problems in financial contracting are supposed to generate the debt–investment–link, now called 'balance–sheet–effect', which Fisher simply assumed. The lower the net worth of business firms, the more severe the adverse selection and moral hazard problems in lending to these firms, hence the higher the agency cost associated with external funding. Agency problems drive a wedge between the cost of uncollateralized external financing and the cost of internal funds which is inversely related to the firm's net worth. It is therefore concluded, that firms with low net worth face higher marginal cost of capital and will *underinvest* compared to firms with high net worth.[2]

This paper argues that the established understanding of the balance–sheet–effect is flawed. First it is shown that agency problems do not generate underinvestment for low net worth firms by virtue of their impact on the shadow cost of external funds. Instead financially constrained firms will underinvest only in projects for which marginal investment is more productive in good states and *overinvest* if marginal investment is more productive in bad states. This insight paves the way for a deeper understanding of the issue. In the underinvestment case marginal investment increases risk while it decreases risk in the overinvestment case. Hence in both cases the firm distorts investment in order to reduce risk. Agency problems of financial contracting, thus explain the formation of risk attitudes at the level of the firm without recurrence to the intrinsic preferences of the economic actors.

The analysis suggests that the increase of the risk–premium which Fisher observed was not coincidental. After experiencing a decline in net worth during the initial economic downturn firms and banks became weary of previously 'reasonable' risk and rebalanced their portfolios

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[1] Others analysts too, were intrigued by the reluctance to finance risky investment, in particular in the banking sector whose balance–sheets had been shattered early on in the crisis (e.g. Friedman & Schwartz (1963), Bernanke (1983)).

[2] See Hubbard (1998), and in particular Bernanke & Gertler & Gilchrist (1998) for a detailed account of the argument. Obviously this balance–sheet–effect has important implications for the transmission of monetary policy and is therefore well covered in modern textbooks on monetary economics e.g. Hubbard (1996), Mishkin (1998).

accordingly. As investment tilted away from risky projects towards those considered safe, marginal returns on risky assets increased while those on safe assets declined. It appears plausible that investment opportunities of the risk increasing type were more abundant, so that increased risk avoidance lead to a decline of aggregate investment — though this cannot be derived from agency theory. Hence, it is the marked increase of the risk spread which indicates that low net–worth and the resulting agency problems accelerated and prolonged the crisis.

This more subtile balance sheet effect has far reaching implications for investment theory and yields interesting empirical predictions, provided that investment can be classified according to its risk–shifting properties. For example inventory investment is likely to be more productive in good states, hence of the risk–increasing type. A well stocked inventory boosts profits when demand is strong and is of little use when demand is weak. Investment which substitutes for labour may be more of the risk–reducing type. Its returns are higher when wages are high which is, everything else equal, bad for profits. Hence, we would expect firms to slash inventories and to increase labour–saving investment in response to financial distress rather than to cut back on all types of investment at the same rate. Agency cost in financial contracting might therefore explain, why certain types of investment (e.g. inventory) tend to vary more strongly over the business cycle than others and why some may even be countercyclical.

Why do agency problems generate risk–aversion? Agency models of financial contracting rest on the assumption that the firm (the agent) cannot be made residual claimant in all states and therefore lacks incentives to act efficiently. The theoretical literature has focussed on four costly options to deal with the problem. Financiers (the principal) may:

1. verify whether the correct action is taken (Townsend (1979), Gale & Hellwig (1985)),
2. punish the firm, e.g. by liquidating assets at a loss (Diamond (1984), Bolton & Scharfstein (1990)),
3. exercise control rights to actively interfere in the firm and to prevent inefficient action (Aghion & Bolton (1992)),
4. accept inefficient behaviour on part of the firm (Sappington (1983).

While these approaches differ widely in focus and modeling strategies they do share important implications: (i) the shadow cost of external funding is increased by the expected agency cost, (ii) optimal contracts feature ex–post deviation from the first best when returns are low — sometimes referred to as 'distortion at the bottom'. It is, however, the second rather than the first feature which provides the key insight into the link between net worth and investment. Ex–post, agency cost are incured when terminal wealth turns out to be low. As investment makes low terminal wealth less likely it helps to save agency cost. This provides an additional return to investment which has to be traded off against the tightening of incentive constraints due to increased external funding. The net impact of investment on expected agency cost is positive or negative depending on whether risk is increased or decreased. Financially constrained firms act as if they are risk averse because they want to reduce expected agency cost. Surprisingly, the details of the agency problem i.e. whether 'distortion at the bottom' is by way of sanctions, through costly interference or verification by financiers, or by inefficient action on part of the firm, are inessential for the impact of net worth on investment. Hence, the modified balance sheet effect is fairly robust.

Since the paper analyzes investment within the framework of optimal financial contracts, it can be related to all of the above mentioned strains of the literature which, however, rarely addresses the issue of investment. An early exception is Gale & Hellwig (1985) who elaborate on Townsend's assumption of 'costly state verification' (CSV) and derive underinvestment for a technology of the risk–increasing type. Bernanke & Gertler & Gilchrist (1998) use this setup as a building block in a dynamic general equilibrium model. And Povel & Raith (2000) obtain underinvestment for risk–increasing investment within a Diamond like model with sanctions. Though their main focus is on comparative statics which turn out to be ambiguous. In Bernanke & Gertler (1989) the scale of investment is fixed, so that investment drops from first–best to zero once net–worth falls below a certain threshold. In the following I consider variable scale investment but I allow for more general investment technologies.

This paper follows the literature mentioned so far, in that the cost of financing always reflect the risk of investment (not just in equilibrium). Essentially, it is assumed that investment is contractible. Hence the firm deviates from first best investment in order to reduce expected agency cost, irrespectively as to which side is formally charged with these cost or how the proceeds are being shared in the optimal contract. This is an important difference to the literature on risk–incentives in the tradition of Jensen & Meckling (1976). There it is argued that the firm distorts non–contractible investment in order to increase risk if its payment profile is convex as it is under debt financing and limited liability (Brander & Lewis (1986)). Similarly, the underinvestment result of Myers (1977) and the overinvestment result of Jensen (1986) are based on the assumption that the firm does not fully take into account the opportunity cost of investment.

Finally, in this paper information is symmetric at the time of contracting but the firm's future actions are not contractible, hence are subject to moral hazard. Mankiw (1986), Bester (1985), Milde & Riley (1988), and de Meza & Webb (1990) draw on selection and signaling models to analyze investment under asymmetric information. This literature has also obtained underinvestment as well as overinvestment, although for reasons very different to those of this paper.

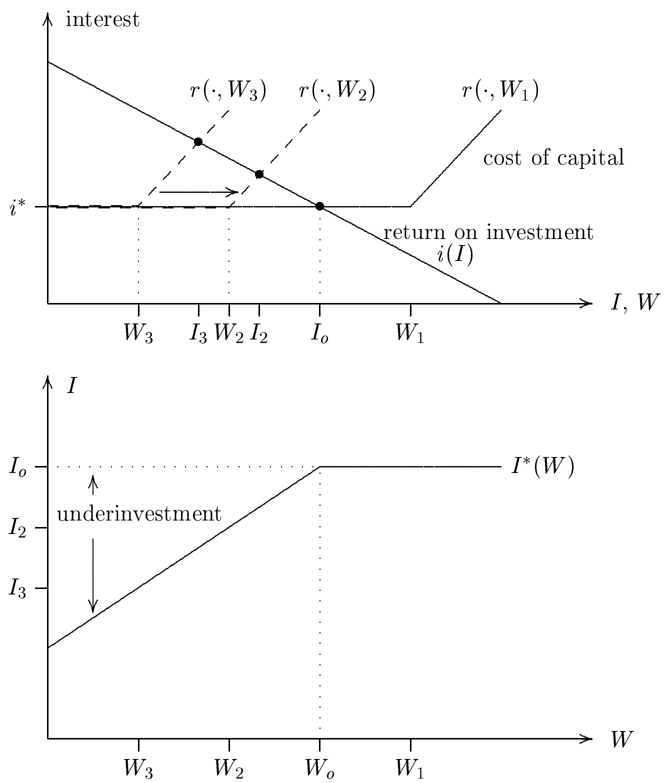
The rest of the paper is organized as follows. In section 2 I restate the popular view of the balance–sheet–effect. Section 3 provides a simple numerical example to illustrate the main point. In section 4 I develop the general framework of the analysis. Sections 5 and 6 characterize optimal financial contracts and optimal investment for two variants of the basic setup. Section 7 summarizes. All proofs are in the the appendix.

2 The Popular Fallacy

If the balance–sheet–effect is to provide a robust building block for monetary economics, the details of the agency problem should not matter for the key link between internal funds or net worth and investment. We therefore restate the conventional view in rather general terms. Hubbard (1998) offers a suggestive explanation of the balance–sheet–effect using a graphic which is reproduced in the upper part of figure 1.[3] The decreasing function $i(I)$ depicts the marginal returns of investment. With a perfect capital market the firm would invest until marginal returns equal the market rate of interest i^*, which would constitute the firm's cost of capital. The firm's net worth (or internal funds) carried over from some previous period, denoted W would be irrelevant for optimal investment I_o.

[3] For a similar exposition see Bernanke & Gertler & Gilchrist (1998).

Figure 1: Firm Wealth and Underinvestment



Now let us assume that uncollateralized external financing suffers from some sort of incentive problem. If the wealth of the firm is increased by one unit and external funding is reduced accordingly, the expected terminal wealth of the firm would increase by more than a unit, due to the saved agency cost. Thus agency cost drive a wedge between the cost of internal and external funds. For investment which can be financed internally or fully secured by collateral ($I < W$) the cost of capital $r(I, W)$ is still given by the market rate of interest. Once the firm can no longer fully collaterilize external funds to avoid agency cost ($W < I$), a premium on external funding increases the marginal cost of capital. Hence, the cost schedule slopes upward to the right of W.

If desired investment is below W, as it happens to be in the case of W_1 (the solidly drawn cost–schedule to the right), a small change of wealth does not effect investment, which would remain at its first best level I_o. If however W falls short of I_o, as it is the case of W_3 (the dashed line to the very left), the mark–up for uncollateralized external funds let the firm *underinvest*. In this situation a small increase of wealth to W_2 shifts the cost of capital schedule from $r(I, W_3)$ to $r(I, W_2)$ and increases investment. To emphasize the empirical implications the lower part of figure 1 displays the resulting relation between investment and the wealth of the firm. When the firm's wealth falls short of the threshold W_o the firm can be considered as being financially constrained. It underinvests $I^*(W) < I_o$ and an improvement in its financial position would increase investment ($dI^*/dW > 0$). Otherwise investment is first best and independent of its financial position ($dI^*/dW = 0$).[4]

As stated the basic mechanism appears very robust. It does not really matter what exactly causes a decline in net worth in the first place. An asset–price deflation, an appreciation of existing debts or a decline in cash–flow drying up internal funds, almost any deterioration of the balance–sheet will lead to reduced investment. Once it is accepted that agency problems raise the shadow cost of external funding, it is difficult to see what could be possibly wrong with the conclusion of underinvestment. However, agency problems not only effect the cost of financing but also the marginal returns on investment. When spending an additional \$ the firm will take into account the impact on expected agency cost, over and above the 'normal' return of investment. It is, therefore, potentially misleading, to translate 'given investment opportunities' into a fixed schedule of marginal returns. Stated in this way, the crucial question appears, whether marginal investment increases or decreases expected agency cost. Only in the former case we should expect underinvestment while in the latter case a reduction of net worth should lead to overinvestment. However, the relation between investment and expected agency cost turns out to be complex. It depends jointly on the way investment shifts the probability distribution of returns and on the features of optimal financial contracts. Before we turn to a formal analysis of the issue, we illustrate the main results with a simple example.

[4] Beginning with Fazzari & Hubbard & Petersen (1988) a number of empirical studies found evidence for a significant positive correlation of investment and cash–flow (a proxy for net wealth) (i.e. $dI^*/dW > 0$) among firms which had been classified a priori as 'financially constrained' (i.e. with $W < W_o$) whereas the null–hypothesis ($dI^*/dW = 0$) could not be rejected for firms classified as 'unconstrained' (i.e. with $W > W_o$). For a review including methodological issues see Hubbard (1998). Kaplan & Zingales (1995) raised doubts concerning the validity of these studies, arguing that within the sample of financially constraint firms originally used by Fazzari & Hubbard & Petersen (1988) it is the group of less constrained firms which exhibit the largest sensitivity of investment to cash–flow. Apart from empirical measurement issues the discussion is about the signs of d^2I^*/dW^2 or possibly dI^*/dW. In this paper we address the more fundamental question of wether $I^*(W) < I_o$.

3 An Example

Consider a poor farmer whose crop depends on uncertain rainfall. If it rains (which happens with probability 4/5) his harvest will be worth 10\$. In case of drought (with probability 1/5) the crop yields nothing. There are two kinds of investment which may also be combined. By spending 5\$ on fertilizer returns can be increased from 10\$ to 20\$, but in the case of draught fertilizer is useless. The farmer may also spend 4.5\$ on drilling a well. If rain is sufficient, irrigation adds nothing, but it secures the yield in case of draught. Abstracting from discounting and assuming risk–neutrality the fertilizer alone generates an expected return of $8\$ = 0.8 \cdot 10\$$, leaving the farmer with a surplus of 3\$. Irrigation in contrast is not worthwhile. Even with fertilizing its expected return is only $4\$ = 0.2 \cdot 20\$$ which does not cover the cost of 4.5\$. In terms of figure 1 fertilizer would be an investment to the left of I_o while drilling a well falls to the right of I_o.

With perfect capital markets the farmer would buy fertilizer and forego irrigation, earning a surplus of 3\$. His initial wealth would not matter. If pennyless, he could always borrow 5\$ for the fertilizer and pay back 6.25\$ if the harvest is good and nothing otherwise. Now assume that external financing suffers from an incentive problem. To be specific, lenders can verify the harvest only at a cost of 20\$. If they do not check up, the farmer can claim crop failure and default even if harvest is in fact good. In order to economize on verification cost, the optimal financial contract will require verification only in the case of default (Townsend (1979)), hence expected cost would amount to $4\$ = 0.2 \cdot 20\$$. To compensate financiers the farmer's expected loan–cost has to increase from 5\$ to 9\$ which exceeds the marginal returns on the fertilizer.[5] With fertilizing being the only option, a poor farmer would rather cut back on investment than tapping the capital market, i.e. he would underinvest to minimize the agency cost. Hence, a decline of the farmer's wealth from 5\$ to zero would be amplified by an additional 3\$–loss of surplus from reduced investment.

But now consider the well again. Taking up a loan of 9.5\$ to buy fertilizer *and* drilling the well generates a safe return of 10\$. Since the loan can always be served, verification cost will never be incurred and the farmer reaps a profit of 0.5\$. The agency problem is still present, as can be seen from the difference in surplus between 3\$ and 0.5\$. Accordingly, the shadow cost of external funding, $12\$ = 9.5\$ + 2.5\$$ exceed the opportunity cost of internal funds, which would be just the 9.5\$ invested. Nevertheless, total agency cost are minimized through *overinvestment*. Instead of cutting back on investment in response to a decline of wealth, the farmer would rather increase investment. This distinct possibility is lost in the graphical illustration presented above, as it is in many verbal explanations of the 'balance–sheet–effect'.

Three tentative conclusions can be derived from this example:

1. Agency problems in financial contracting do not generate a positive link from net worth to investment (the underinvestment hypothesis) by virtue of their impact on the shadow cost of external funds.

2. Instead it is the impact of additional investment on expected agency cost which determines whether there is underinvestment or overinvestment.

3. As a result the bias in investment appears to depend largely on assumptions about technology, on which the literature is usually silent.

[5] In order to recoup a loan of 5\$ and the expected verification cost of 4\$, the lender has to charge $11.25\$ = 6.25\$ + 5\$$ if the harvest is good, which is more than the 10\$ increase in yield.

For a more constructive hint, recall that fertilizing is only productive in good states whereas irrigation is only productive in bad states. Hence investment in fertilizer increase the volatility of returns by increasing losses in bad states and increasing profits in good states. Investment in irrigation in contrast reduces risk. Since we obtained underinvestment for fertilizer and overinvestment for irrigation, it appears as if agency cost distort investment so as to reduce risk — in spite of all agents being risk–neutral. In the remainder of the paper we generalize these results to a much larger class of agency problems and technologies.

4 The Framework

Assumptions about technology and uncertainty are often kept at the background. Since they will turn out to be of imminent importance, whereas the details of the agency problem are not, we put them at the beginning of our formal analysis. The relation between investment I and uncertain *gross return* $\pi \equiv [\underline{\pi}, \bar{\pi}]$ is conveniently described by the probability function $F(\pi, I)$ and its density $f(\pi, I)$. An increase of I improves F in the sense of first order stochastic dominance with a decreasing rate of return, $F_I < 0$, $F_{II} > 0$. Expected *net return* is given by

$$\mathrm{E}[\pi - I] = \int_{\underline{\pi}(I)}^{\bar{\pi}(I)} \pi f(\pi, I)\, d\pi - I = \bar{\pi}(I) - \int_{\underline{\pi}(I)}^{\bar{\pi}(I)} F(\pi, I)\, d\pi - I,$$

and the first order condition for I_o to maximize expected net return simplifies to $1 = \int_{\underline{\pi}}^{\bar{\pi}} -F_I \, d\pi$. To develop the intuition for later results let us briefly introduce a random variable z and make explicit the dependence of π on I by looking at $\pi(I, z)$. As a matter of convention a larger z shall indicate better states of nature e.g. $\pi_z > 0$. First, we establish the relation between the the monotony of $-F_I/f$, the dependence of marginal return to investment on the state of nature i.e. the sign of π_{zI}, and the risk–shifting properties of I.

LEMMA 1

i $-F_I/f$ *is increasing (decreasing) in* π *if and only if marginal return on investment is higher (lower) in good states of nature, i.e.* $\pi_{zI} > 0\,(< 0)$.

ii If $-F_I/f$ *is increasing in* π*, then a marginal increase of investment* I *(evaluated at its first best level* I_o*) increases the risk of net returns in the sense of a mean preserving spread (MPS). If the slope of* $-F_I/f$ *is negative, then risk is decreased by a mean preserving compression (MPC).*

Lemma 1 provides us with a convenient description of different types of technologies, closely resembling the distinction between fertilizer and irrigation in our previous example. Since investment increases risk if $d(-F_I/f)/d\pi > 0$ (or equivalently $\pi_{zI} > 0$) we will refer to this assumption as the MPS–case. We speak of the MPC–case if the signs are reversed and risk is reduced.[6]

[6]It is easy to find examples for both types of investment. For the MPS–case consider the commonly used concave production function $y(K, L)$ with inputs capital K and labour L. For simplicity take the number of workers as fixed. Investment determines the capital stock ($K = I$) and is selected given expectations over a random price p for output. From the ex–post profit function $\pi(I, p) = p \cdot y(I, L) - wL$ it follows that $\pi_p > 0$ and $\pi_{pI} > 0$. Hence marginal investment is more productive in good states, as is required for the MPS–case. Obviously any multiplicative shock to productivity would yield the same result. To obtain the

Having described the investment opportunities of the firm, we now turn to the financial side, which is where the agency problem comes in. The firm is risk–neutral and issues securities to raise funds from financiers, also risk–neutral, in a deep and competitive capital market. There are no a–priori restrictions on the design of the securities. Hence the payout to external financiers as described by a state contingent 'share' $s(\pi)$ of the project's return, will be part of the optimization problem of the firm.

After the investment is made the firm observes π and can 'appropriate' part of it leaving only $x \leq \pi$ to be shared.[7] External funding is burdened with an agency problem because neither π nor appropriation itself is contractible. Hence, any provisions which depend on π have to be incentive compatible.[8] In one form or the other this is a standard assumption in the principal agent literature on financial contracting. Since we are suggesting a general framework, we stay deliberately vague about the nature of 'appropriation'. Realized return x may fall short of potential value π due to theft, fraud, consumption on the job, lack of effort, or the waste of resources on pet projects and empire building.[9] Accordingly, there are many ways to model the firm's gain from appropriation. Here we assume that appropriation involves a loss compared to a straight pay. To keep the analysis simple, let the function $a(\pi - x)$ measure cost of appropriation by the difference between the wealth diverted $\pi - x$ and the payment which would leave the firm equally well–off. The net gain from appropriation is then $\pi - x - a(\pi - x)$. Since 'appropriation' is used as a catch all notion for any behaviour on part of the firm which is inefficient, the interpretation of a depends on the nature of its activity. In case of theft and fraud, it stand for the cost of falsifying income reports, hiding transfers, bribing employees etc. When effort is too low, it measures the difference between lost output and the money value of the disutility of effort. If consumption on the job is the issue, it is the difference between corporate money spend and the private willingness to pay. If decision–makers in the firm continue with unprofitable lines of business to avoid conflict in the team, it is the difference between lost profits and what they would have been prepared to spend out of their own pocket to prevent the row.[10] The size of appropriation cost provides a measure for the severity of the agency problem. The higher the appropriation cost are, the lower are the firm's gains from appropriation and the less inclined it will be to act detrimental to the interest of the financiers.[11]

MPC–case assume that output–price is certain while wage w is random and that investment reduces labour cost without increasing capacity. Ex–post profit is given by $\pi(I, w) = p \cdot y(K, L) - wL \cdot c(I)$ with $c'(I) < 0$. High wages correspond to low profits ($\pi_w < 0$). Since $\pi_{wI} = -c'L > 0$ marginal return on investment is higher in bad states.

[7] The possibility of faking good results, $x > \pi$, is ignored in order to streamline the exposition. Since the firm has no incentive to do so at the optimal contract, the results would not change.

[8] Applying the *revelation principle*, we analyse the contracting problem as a *direct* incentive compatible mechanism. $s(\cdot)$ as any other state contingent instruments of the financial contract depend directly on the unobservable variable as it is revealed by the firm. The incentive constraint requires that the contract is designed to ensure that conveying the true π is in the best interest of the firm.

[9] Most examples given in the literature relate to the problem of motivating professional managers. Similar issues arise when an investor is expected to monitor and control the management. Since part of any increase in firm value accrues to other claimants he may exert too little effort. In addition he may abuse its sway over the management to fleece the company e.g. by manipulating the terms for transactions with more fully owned firms.

[10] Much of the literature on security design assumes that the firm is indifferent between 'appropriated resources' $(\pi - x)$ and 'straight pay' $(x - s(\pi))$. While this assumption appears highly implausible, it can be obtained as a special case by setting $a = 0, \forall x \leq \pi, \pi$.

[11] In this paper $a()$ is assumed to be exogenous. We do not take into account measures to alleviate the incentive problem by making appropriation difficult. These may include constraints imposed by the corporate charter or by organizational design, monitoring, enforcement of strict accounting principles, restrictions on ownership stakes with business–partners, covenants restricting capital expenditures or requiring the sale of

The firm's liquid wealth is denoted W. To finance investment I the firm raises $I - W$ externally. Given risk–neutrality and abstracting from discounting, the expected payments to financiers must not be less than the value of their contribution (participation constraint):

$$\int_{\underline{\pi}}^{\bar{\pi}} s(\pi) f(\pi; I)\, d\pi \geq I - W \tag{1}$$

When final payments are made the firm has no liquid wealth left apart from the returns of investment. Hence payments to financiers are restricted according to (wealth constraint):

$$s(\pi) \leq \pi - a(\pi - x(\pi)). \tag{2}$$

In addition to liquid wealth the firm might possess illiquid assets which can be transferred only at a loss. Let $L(\pi)$ measures the expected loss inflicted on the firm in state π through partial or probabilistic seizure of illiquid assets. To simplify the exposition, we omit any gains the financiers might receive from liquidation. In this case illiquid assets are worthless as collateral but they still might serve to discipline the firm. For simplicity we refer to $L(\pi)$ as the probability of firm liquidation.[12]

In its most general form the financial contract sets investment I, a state contingent payment schedule $s(\pi)$, a liquidation schedule $L(\pi)$ and a schedule of realized returns $x(\pi)$. The timing is as follows:

1. the contract determines I, $s(\pi)$, $L(\pi)$, and $x(\pi)$,
2. financiers deliver $I - W$ and investment takes place,
3. the firm observes π and announces $\tilde{\pi}$,
4. based on the firms announcement s, L and x are implemented.[13]

The first–best contract (with π being contractible) would require efficient action on part of the firm $x(\pi) = \pi$ in all states and never seize illiquid assets $L(\pi) = 0$. Avoiding the cost associated with appropriation and the loss from liquidation any payment schedule s with $s(\pi) \leq \pi$ could be used to fulfill the financiers' participation constraint. However, if π is not contractible such a contract is generally not incentive compatible. Define the firm's payoff when revealing π truthfully as

$$w(\pi) \equiv \pi - s(\pi) - a(\pi - x(\pi)) - L(\pi). \tag{3}$$

In addition we define its payoff as a function of the true state π and its message $\tilde{\pi}$ (possibly false): $\tilde{w}(\pi, \tilde{\pi}) = \pi - s(\tilde{\pi}) - a(\pi - x(\tilde{\pi})) - L(\tilde{\pi})$. The firm will reveal π truthfully provided the contract fulfills the following incentive compatibility constraint:

$$w(\pi) \equiv \tilde{w}(\pi, \pi) \geq \tilde{w}(\pi, \tilde{\pi}), \quad \forall \quad \pi, \tilde{\pi} \leq \pi. \tag{4}$$

assets, external evaluation or active intervention by financiers. Since all these measures are costly, a itself will be the result of some sort of optimization.

[12] Our results do not depend on any particular interpretation of L nor would they change much if financiers were to gain or loose from liquidation. For example Diamond (1984) suggests that firm owners can be punished through the time lost in bankruptcy procedures. In this case we would expect, creditors to incure a loss as well. The firm's earning capabilities may also be disrupted through a high burden of senior financial claims which make refinancing of future operations difficult.

[13] We do not address the issue of renegotiation in this paper and assume that the contracting parties can commit to outcomes which are ex–post inefficient.

It ensures that the firm's final wealth when telling the truth, $\tilde{\pi} = \pi$ (the left side) is not smaller than when falsely claiming a lower $\tilde{\pi} < \pi$ (the right side).

When issuing financial claims the firm will maximize the expected value of w subject to the constraints stated above. Formally, it solves:

PROGRAM 1.

$$\max_{s,x,I,L} \int_{\underline{\pi}}^{\bar{\pi}} (w(\pi) + \lambda s(\pi)) f(\pi, I)\, d\pi - \lambda(I - W)$$

subject to: (2) and (4),

where λ, the Lagrange–multiplier associated with the participation constraint of the financiers (1), yields the shadow cost of external funds. The participation constraint for the firm has been dropped for convenience. Throughout the following it is assumed that a solution satisfying both reservation utilities exits, and that the incentive constraint (4) is binding. In the next two sections we will characterize optimal securities and investment for two variants of this problem.

5 Costly Liquidation and Investment

In this section we focus on a variant of the agency problem in which the optimal financial contract solves the incentive problem through costly liquidation. With respect to appropriation cost we assume that the loss is proportional to the amount of resources appropriated:

ASSUMPTION 1 $a(\pi - x) = \alpha \cdot (\pi - x)$, $\alpha \in [0, 1]$.

The exogeneous loss–factor α indicates how well aligned the interest of the firm and its financiers are, i.e. how severe the agency problem is. For $\alpha = 0$ the firm can appropriate resources very efficiently. It has therefore strong incentives to act contrary to the interests of its financiers. For $\alpha = 1$ in contrast the interest of both sides are well aligned and the incentive constraint would no longer be binding at the optimal contract.

We characterize the optimal contract in several steps. First, we look at realized firm value x^*, then we show that the optimal choice of the payment schedule s^* and the liquidation function L^* is characterized by one parameter D, which can be interpreted as the face value of risky debt, while the exogenous parameter α yields the fraction of outside equity. In the final step, we address investment I^* assuming that the firm cannot finance I_o without raising risky debt.

With proportional cost it is always better to raise payment to security holders rather than to allow for inefficient action on part of the firm. Hence, the optimal contract completely avoids appropriation by setting x to its first best value.

PROPOSITION 1 *With a proportional loss, optimal appropriation is zero in all states: Assumption 1* $\Longrightarrow x^*(\pi) = \pi$, $\forall \pi$.

This partial result reveals how restrictive the assumption of proportional cost really is. As we will show below the optimal contract features costly liquidation and distort investment, but it does not depart a whit from the first best action in any possible state. The following proposition establishes the debt–equity–feature of the optimal financial contract.

PROPOSITION 2 *For any given I, the optimal contract resembles a combination of debt with face value D and equity. As π falls below the threshold of D the probability that the firm goes into liquidation increases and financiers obtain the whole output. For π larger than D no liquidation occurs and financiers recover a fixed payment equal to D plus a share α of the residual $\pi - D$. Formally:*
Assumption 1 $\Longrightarrow$ if $\{s^*, \beta^*\}$ solves program 1, then $\exists\ D \geq 0$ such that:

$$s^*(\pi) = \begin{cases} \pi; & \pi < D \\ D + \alpha(\pi - D); & \pi \geq D \end{cases}$$
$$L^*(\pi) = \begin{cases} (1-\alpha)(D-\pi); & \pi < D \\ 0; & \pi \geq D. \end{cases}$$

Three features of this contract invite the interpretation in terms of risky debt and outside–equity: (i) the firm is liquidated only if a fixed, hence debt–like, claim cannot be served and the probability of liquidation increases as the shortfall gets larger. (ii) In case of 'insolvency' financiers recover as much as possible. (iii) In good states the firm stays alive. Financiers obtain their fixed claim plus a constant, hence equity–like, share α of the residual. The debt–equity contract minimizes the cost of liquidation for any given expected payment to the financiers.[14] The firm retains as much inside equity as is necessary to provide it with incentives for efficient action in states of solvency. Only when this is not possible (in states of insolvency), would costly sanctions be applied to prevent the firm from strategic default.[15]

Since α is exogenously given, the optimal contract maintains a fixed relation between inside and outside equity. Hence, risky debt is the marginal source of finance and the firm's leverage is determined by the amount of external finance needed. Using propositions 1 and 2 we can rewrite program 1 to obtain:

PROGRAM 2.

$$\max_{D,I} \quad \int_{\underline{\pi}}^{\bar{\pi}} (1-\alpha)(\pi - D) f(\pi; I)\, d\pi$$
$$+ \lambda \int_{\underline{\pi}}^{D} \pi f(\pi; I)\, d\pi + \lambda \int_{D}^{\bar{\pi}} (D + \alpha(\pi - D))\, f(\pi; I)\, d\pi - \lambda(I - W)$$

Recall that first–best investment I_o solves $1 = \int_{\underline{\pi}}^{\bar{\pi}} -F_I\, d\pi$. The following proposition characterizes λ^* and I^* for the case that the firm has to issue risky debt to finance investment i.e. $D^* > \underline{\pi}$.

PROPOSITION 3
(i) Agency cost drive a wedge between the cost of internal and external funds, i.e. the shadow

[14] Obviously, proposition 2 keeps us close to the established literature on optimal financial contracts. In particular, we obtain Diamond's (1984) optimality of pure debt–financing as the limiting case for $\alpha \to 0$.

[15] While s can be conceived as a combination of debt and equity, there is nothing in the model to require that s should in fact be divided into distinct claims, debt and equity, that payment in the control region accrues exclusively to debt–holders, or that residual control rights should be attached in any particular manner to these securities. This lack of explanatory power, however, should not be seen as a weakness of the framework — quite to the contrary. The underlying assumption that commitment to an ex–post inefficient allocations is possible can now be justified by arguing that these additional features can be used to prevent renegotiation and motivate a group of financiers to carry out the 'punishment'. Among recent papers explaining these additional features in this spirit are Berglöf & von Thadden (1994), Bolton & Scharfstein (1996) and Dewatripont & Tirole (1994).

price of external funds is larger than one, $\lambda^* = 1/(1 - F(D^*)) > 1$.
(ii) The optimal investment I^* *solves:*

$$1 = -\int_{\underline{\pi}}^{\overline{\pi}} F_I \, d\pi - (1 - \alpha)\gamma(I, D), \tag{5}$$

where $\gamma(I, D)$ *is defined as:*

$$\gamma(I, D) \equiv -F(D; I) \int_{\underline{\pi}}^{\overline{\pi}} F_I \, d\pi + \int_{\underline{\pi}}^{D} F_I \, d\pi. \tag{6}$$

(iii) There will be underinvestment in the MPS–case and overinvestment in the MPC–case: $d(-F_I/f)/d\pi \gtreqless 0 \Longrightarrow I^* \lesseqgtr I_o$.

According to equation (5) investment is optimal if marginal cost of investment (the left hand side), equates the expected total marginal return (the right hand side). The latter, however, consists of two terms. The first term stands for the 'normal' return which would determine investment in the absence of agency problems. The second term provides for an adjustment which reflects the subtle interaction of investment with ex post distortions. For $\gamma(I, D) > 0$ investment increases expected agency cost, hence, marginal returns on investment have to be higher than in the first best case, which in turn requires underinvestment by second order condition. However, for the same reason $\gamma < 0$ implies overinvestment. As mentioned above, the agency problem disappears as $\alpha \to 1$ for which investment would approach the first best level I_o.

Equation (6) reveals that opposing forces determine γ. On the one hand, higher investment demands higher expected payouts to financiers and this increases the distortions necessary to fulfill the incentive constraint. This is the effect on which the naive underinvestment hypothesis rests. It shows up in the first term of (6) which is always positive, as required for underinvestment. The second term, however, is always negative pushing towards over-investment. Investment increases returns in states of default which decreases the expected loss from liquidation.

The relative strength of the effects depends on investment technology. When investment is more productive in good states and risk is increased by higher investment (MPS–case), the underinvestment effect dominates. However, the overinvestment effect is stronger if investment is more productive in bad states and risk is reduced by higher investment (MPC–case). In both cases the firm distorts investment in order to reduce risk. Hence, when net worth is so low that first best investment I_o can only be financed by accepting the agency cost associated with risky debt the firm acts as if being risk–averse.

6 Inefficient Action and Investment

While being more general than zero cost of appropriation, the assumption of a proportional loss invites further generalization. Also, we know from proposition 1 that non–linear cost are necessary for optimal contracts to compromise on first best action. Presumably, 'on the job consumption', 'empire–building', perks and pet–projects suffer from decreasing marginal utility. Hence, we assume appropriation cost to be convex. Formally, we replace assumption 1 by the following assumptions on $a(\pi - x)$:

Assumption 2 $a'(0) = 0, \quad 0 \le a' < 1, \quad a'' > 0, \quad a''' \le 0 \ \ \forall x \le \pi.$

With $a' < 1$ the firm is never satiated with appropriation. $a''' \leq 0$ implies that a''/a' is decreasing which is needed for second order conditions. In addition we require $1 - F$ to be log–concave (or equivalently the hazard rate $f/(1-F)$ to be non–decreasing). To focus on possible deviations from first best action, we rule out the use of sanctions, $L = 0$, e.g. because the firm lacks illiquid assets. The remaining instruments of the contract are I, $s(\pi)$ and $x(\pi)$, and the firm's payoff simplifies to

$$w(\pi) \equiv \pi - s(\pi) - a(\pi - x(\pi)). \tag{7}$$

Problems of this type have been well studied in the principal agent literature. A convenient solution method is due to Guesnerie & Laffont (1984) (for simple exposition see Laffont (1989)). The incentive constraint (4) is replaced by a first order condition and a monotonicity constraint. The latter ensures that second order condition are fulfilled. The wealth constraint (2) is translated into a boundary condition for the lower bound. With these transformations a comprehensive characterization of the solution can be obtained through the techniques of optimal control theory. To save space we ignore the boundary condition and use a simpler approach which is somewhat less rigorous and powerful but sufficient for our limited purpose here.[16]

Recall from the definitions of w and $\tilde{w}$ that $w(\pi) \equiv \tilde{w}(\pi, \pi)$, hence $w'(\pi) = \tilde{w}_\pi(\pi, \pi) + \tilde{w}_{\tilde{\pi}}(\pi, \pi)$. The first order condition for truth–telling being optimal is $\tilde{w}_{\tilde{\pi}}(\pi, \pi) = -s' + a'x' = 0$. It is fulfilled if s and x are chosen, so that firm wealth increases according to the following incentive constraint

$$w'(\pi) = \tilde{w}_\pi(\pi, \pi) = 1 - a'(\pi - x(\pi)), \quad \forall \pi \tag{8}$$

Integration gives the firm's wealth as $w(\pi) = \int_{\underline{\pi}}^{\pi} (1 - a') d\tau$ and expected wealth as $\int_{\underline{\pi}}^{\bar{\pi}} \int_{\underline{\pi}}^{\pi} (1 - a') d\tau f \, d\pi$. Partial integration and rearranging yields a condition for expected wealth which incorporates the first order condition for incentive compatibility[17]

$$\int_{\underline{\pi}}^{\bar{\pi}} w(\pi) \, f d\pi = \int_{\underline{\pi}}^{\bar{\pi}} \frac{1-F}{f} (1 - a') \, f d\pi. \tag{9}$$

Using (7) and (9) to substitute $s(\pi)$ and $w(\pi)$ in program 1 we obtain:

PROGRAM 3.

$$\max_{I,x} \int_{\underline{\pi}}^{\bar{\pi}} \left[\lambda(\pi - a) + (1 - \lambda) \frac{1-F}{f} (1 - a') \right] f \, d\pi - \lambda(I - W)$$

Again, we derive the solution to this problem in several steps. First, we ask how appropriation is optimally chosen. Then we characterize the payment schedule and finally we turn to the issue of investment.

[16] It can be shown, that $\lambda^* > 1$ when the boundary condition binds and $\lambda^* = 1$ otherwise. Here we simply assume that the former is the case.

[17] The second order condition requires $\tilde{w}_{\tilde{\pi}\tilde{\pi}}(\pi, \pi) < 0$. Since the first order condition assumes the status of an identity we have $\frac{d}{d\pi} \tilde{w}_{\tilde{\pi}}(\pi, \pi) = \tilde{w}_{\tilde{\pi}\tilde{\pi}}(\pi, \pi) + \tilde{w}_{\tilde{\pi}\pi}(\pi, \pi) = 0$. Hence, the second order condition requires $\tilde{w}_{\tilde{\pi}\pi}(\pi, \pi) = a''(\pi - x(\pi))x'(\pi) \geq 0$. It is fulfilled provided that $x' \geq 0$. As the next proposition shows this monotonicity constraint is not binding at the optimal solution. It may therefore be dropped from the optimization problem.

PROPOSITION 4 *With convex cost, optimal appropriation* $(x^* - \pi)$ *is positive (except for* $\bar{\pi}$*) but strictly decreasing as returns improve. Formally:*
Assumption 2 $\Longrightarrow x^*(\bar{\pi}) = \bar{\pi}$ *and for* $\pi < \bar{\pi}$*:*

$$x^*(\pi) < \pi, \quad \text{and} \quad \frac{dx^*}{d\pi} > 1,$$

and $x^*(\pi)$ *is implicitly defined by*

$$f(\pi)a'(\pi - x)\lambda = (1 - F(\pi))a''(\pi - x)(\lambda - 1). \tag{10}$$

The contract is shaped by the following trade–off: High payouts are needed in order to fulfill the participation constraint. Since the wealth constraint prevents payouts to be high in bad states they have to be raised as returns improve. According to the incentive constraint this can only be achieved by tolerating appropriation. Suppose we increase appropriation at π marginally holding the firm's expected wealth constant. The expected loss would be fa' times the shadow price of funds, λ. By the incentive constraint (8) this would allow for an increase of payouts by a'' not just for π but for all better realizations, in probability $(1-F)$. The net–value of these transfers is $(\lambda - 1)$, the difference between shadow value of external and internal funds. Equation (10) states that x will be chosen to equate marginal cost of appropriation to marginal benefits of the resulting increase of payouts. It can be rewritten as

$$\frac{f(\pi)}{1 - F(\pi)} = \frac{a''(\pi - x)}{a'(\pi - x)} \cdot \frac{\lambda - 1}{\lambda}.$$

Assumption (2) implies that the right hand side is decreasing in x for any given π. By log–concavity of $(1-F)$ the left hand side decreases monotone towards zero. The higher the returns, the less likely it is to receive an even higher return. Hence, the gains from allowing x to drop below its first best level are smaller the higher π is. As in the previous section the optimal contract features distortions which are stronger the worse the state of nature is.

The claims in proposition 4 relate observable variables, payments s and realized returns x, to potential firm value π which is by assumption not observable. In order to compare the results to observable features of financial contracts we characterize the payment schedule in terms of x (slightly abusing notation):

COROLLARY 1 *Payments to both sides are strictly increasing in observed returns, those to the manager are convex, (hence those to the financiers are concave):*

$$s'(x) = a'(\pi - x), \quad 0 < s' < 1, \quad s''(x) < 0$$

Empirical payment profiles are typically piece–wise linear whereas the solution derived here is a smooth curve. Nevertheless, its overall shape fits empirical financing patterns reasonably well. Since a combination of debt, preferred stock and ordinary shares allow for a better approximation of s than debt and equity only, the model offers in fact an explanation of the use of richer financial instruments.

Our last proposition characterizes again the optimal investment.

PROPOSITION 5 *If the incentive constraint binds at the optimal contract* $(\lambda > 1)$*, investment is distorted away from its first best level:*

(i) I^* solves:

$$1 = -\int_{\underline{\pi}}^{\overline{\pi}} F_I \, d\pi - \frac{\lambda - 1}{\lambda} \int_{\underline{\pi}}^{\overline{\pi}} (1 - a')\gamma(I) f \, d\pi, \tag{11}$$

with

$$\gamma(I) \equiv \frac{\partial}{\partial I}\left(\frac{1-F}{f}\right) - \frac{F_I}{f}\frac{\partial}{\partial \pi}\left(\frac{1-F}{f}\right).$$

(ii) There is underinvestment in the MPS–case and overinvestment in the MPC–case: $d(-F_I/f)/d\pi \gtreqless 0 \Longrightarrow I^* \lesseqgtr I_o$.

The first order condition for optimal investment (11) resembles equation (5) of proposition 3. Investment is again optimal if the marginal cost of investment equate the expected marginal return which includes a term reflecting the impact of investment on agency cost. As before the sign of the agency–term depends on a function γ and is ambiguous. The first term of $\gamma(I)$ is positive, provided investment improves the distribution of F in the sense of hazard rate dominance which is slightly stronger than first order stochastic dominance. It tends to reduce investment since capital cost are inflated by the efficiency loss due to poor incentives. On the other hand, investment shifts the distribution towards good states for which these losses are low. This effect pushes towards more investment. It shows up in the second term of γ, which is negative by $F_I < 0$ and $\frac{\partial}{\partial\pi}[(1-F)/f] < 0$. Again the combined effect can be determined only in cases for which $-F_I/f$ is monotone in π. And again the firm will underinvest or overinvest depending on which is necessary to decrease the volatility of returns.

7 Conclusion

So far the main result that agency cost of external funding impair the firms' ability to take investment risk has been formally derived for two variants of the agency problem. To save space I will not prove that it extends to the other cases mentioned in the introduction. Gale & Hellwig (1985) allow financiers to verify the true state at some cost (CSV) and derive underinvestment for the MPS–case. The example in section 3 should convince the reader that overinvestment will prevail in the MPC–case, and Hubert (1999) proves the claim in a model in which financiers can raise the cost of appropriation by active interference. Hence I feel confident to summarize the results in rather general terms.

If the firm's net worth is insufficient to avoid the agency cost associated with financial contracting, external financing carries a premium over the use of internal funds. However, contrary to the common presumption, such a wedge between the cost of internal and external funds is not sufficient to generate underinvestment. The reason is that optimal financial contracts feature costly distortions when returns are low in order to fulfill incentive constraints. As investment decreases the probability of low returns it also helps to reduce expected agency cost. When agency problems matter, a dollar raised from external funds costs more than one dollar, but a dollar earned in bad states also yields more than one dollar. Hence, it is a–priori not clear whether we should expect underinvestment or overinvestment for firms with low net worth.

The paper derives a simple condition on investment technology which resolves the ambiguity. The net effect of marginal investment on expected agency cost is positive if investment is more productive in *good* states. In this case a firm with low net worth will underinvest. If, however investment is more productive in *bad* states, then increasing investment alleviates the agency problem. In this case there will be overinvestment. It is therefore investment technology, and not the shadow cost of external funds or the details of the agency problem, which determines whether there is overinvestment or underinvestment.

A robust though more subtile 'balance sheet effect' was obtained by looking at the risk–shifting properties of investment. As the firm becomes dependent on uncollateralized external funds it distorts investment in order to reduce the volatility of returns. Hence, low net worth firms and banks will shun reasonable risk, by underinvesting in risky projects and overinvesting in safe projects. This result lends credibility to widely held believes: (i) that entrepreneurial wealth (inside equity) facilitates risky investment and (ii) that risk tolerance is abnormally low during major economic recessions.

8 Appendix

Proof of Lemma 1

(i): Take the total differential of $F(\pi(z,I),I)$ to obtain

$$dF(\pi(z,I),I) = f \cdot (\pi_z dz + \pi_I dI) + F_I dI$$

Note that $dz = 0 \implies dF = 0$, hence

$$\pi_I = -\frac{F_I}{f}, \quad \text{and} \quad \pi_{Iz} = -\frac{d}{d\pi}\left(\frac{F_I}{f}\right) \cdot \pi_z,$$

which proves the claim because $\pi_z > 0$.

(ii): Let $G(r,I) = F(r+I,I)$ denote the distribution of net–return $r = \pi - I$. The optimal choice of I_o requires

$$0 = \int_{\underline{r}}^{\bar{r}} G_I(z,I_o)\,dz = \int_{\underline{r}+I}^{\bar{r}+I} (f + F_I)\,dz = \int_{\underline{\pi}}^{\bar{\pi}} (1 + \frac{F_I}{f}) f\,dz.$$

Hence, for I_o and any $\hat{r}$ with $\underline{r} < \hat{r} < \bar{r}$

$$\frac{d}{d\pi}\left(\frac{F_I}{f}\right) > 0 \quad \forall \pi \quad \implies \quad \int_{\underline{r}}^{\hat{r}} G_I\,dz = \int_{\underline{r}+I}^{\hat{r}+I} \left(1 + \frac{F_I}{f}\right) f\,dz < 0.$$

According to the inequality on the right side, an increase of I improves G in the sense of second order stochastic dominance. Together with the first order condition we may therefore conclude that marginal investment evaluated at I_o reduces the risk of net–return by a mean preserving compression (MPC) for $d(F_I/f)/d\pi > 0$. The proof for the MPS–Case is analog. □

Proof of Proposition 1

We show that any feasible contract with $x(\pi) < \pi$ can be replaced by another contract with $\hat{x}(\pi) = \pi$ which fulfills all constraints and leaves financiers better off without making the firm worse off. Using assumption 1 the incentive constraint (4) can be rewritten as:

$$\begin{aligned} \pi - s(\pi) - \alpha\cdot(\pi - x(\pi)) \quad \geq \quad & \tilde{\pi} - s(\tilde{\pi}) - \alpha\cdot(\tilde{\pi} - x(\tilde{\pi})) \\ & - (L(\tilde{\pi}) - L(\pi)) + (1-\alpha)(\pi - \tilde{\pi}) \end{aligned}$$

The wealth constraint (2) requires the expressions in the first line to be non–negative. Consider a feasible contract $\{s, x, L\}$ featuring $x(\pi) < \pi$ on a non–degenerated subset. Replace this contract through a contract with: $\hat{x}(\pi) = \pi$ and $\hat{s}(\pi) = s(\pi) + \alpha\cdot(\pi - x(\pi))$. On substituting this contract in the equation above it is easy to see that $\{\hat{s}, \hat{x}, L\}$ also fulfills the incentive and wealth constraints. It is strictly preferred by financiers, since pay–outs are larger in some states and never smaller. The firm is indifferent because payments to financiers are raised only to the extend that appropriation cost are reduced. Hence $\{\hat{s}, \hat{x}, L\}$ is Pareto–superior. □

Proof of Proposition 2

With the partial result of proposition 1 program 1 simplifies to:

PROGRAM 4.

$$\max_{s(\pi),L(\pi),I,\lambda} \int_{\underline{\pi}}^{\bar{\pi}} [\pi - (1-\lambda)s(\pi) - L(\pi)] f(\pi, I)\, d\pi - \lambda(I - W) \qquad \text{(V.4)}$$

$$s(\pi) \leq s(\tilde{\pi}) + \alpha(\pi - \tilde{\pi}) + (L(\tilde{\pi}) - L(\pi)) \quad \forall\, \pi, \tilde{\pi} < \pi \qquad \text{(IC.4)}$$

$$s(\pi) \leq \pi \qquad \forall\, \pi \qquad \text{(WC.4)}$$

Suppose there exist a contract $\{\tilde{s}, \tilde{L}\}$ for which $\tilde{s}$ is different from s^* on a subset $\tilde{\Pi}$ with $\int_{\tilde{\Pi}} dF \neq 0$. Since the wealth constraint binds for $\pi \leq D$, there exist an $\pi_0 \in [D, \bar{\pi}]$ for which $\tilde{s}(\pi_0) > s^*(\pi_0)$, otherwise the participation constraint would be violated. To obey the incentive constraint, however, the probability of termination has to be raised by at least $(\tilde{s}(\pi_0) - s^*(\pi_0))$ for $\pi \in [\underline{\pi}, D]$. While $\tilde{L} > L^*$ on $[\underline{\pi}, D]$ it cannot be lower on $[D, \bar{\pi}]$. Hence, the expected cost of disruption is strictly higher with $\{\tilde{s}, \tilde{L}\}$. Since the expected payments to financiers cannot be smaller, the contract is inferior. □

Proof of Proposition 3

The claim concerning λ^* follows immediately from the first order condition for the optimal choice of D^*.

Partial integration of the first order condition for I^* yields:

$$\begin{aligned} 0 &= -(1-\alpha)\int_{\underline{\pi}}^{\bar{\pi}} F_I\, d\pi - \lambda\left(\int_{\underline{\pi}}^{D} F_I\, d\pi + \alpha\int_{D}^{\bar{\pi}} F_I\, d\pi + 1\right) \\ &= -1 - \int_{\underline{\pi}}^{\bar{\pi}} F_I\, d\pi + (1-\alpha)\left(\int_{\underline{\pi}}^{D} F_I\, d\pi - (1/\lambda)\int_{\underline{\pi}}^{\bar{\pi}} F_I\, d\pi\right) \end{aligned}$$

Substitution with $\lambda = 1/(1 - F(D, I)$ and rearranging yields equation (5), with $\gamma(I, D)$

being defined as:

$$\begin{aligned}\gamma(I,D) &\equiv (1-F(D,I))\int_{\underline{\pi}}^{\bar{\pi}} F_I\,d\pi - \int_D^{\bar{\pi}} F_I\,d\pi \\ &= -F(D,I)\int_{\underline{\pi}}^{\bar{\pi}} F_I\,d\pi + \int_{\underline{\pi}}^{D} F_I\,d\pi.\end{aligned}$$

Now we show that sign $\gamma =$ sign $\frac{d}{d\pi}[-F_I/f]$. Since

$$\text{sign }\gamma(I,D) = -\text{sign}\left(-\int_{\underline{\pi}}^{\bar{\pi}} F_I\,d\pi - \frac{-\int_D^{\bar{\pi}} F_I\,d\pi}{1-F(D,I)}\right)$$

and $\gamma(I_o,\underline{\pi}) = 0$, it is sufficient to show that

$$\frac{d}{d\pi}\left(\frac{-F_I(\pi)}{f(\pi)}\right) \gtrless 0,\ \forall\pi \quad \Longrightarrow \quad \frac{d}{dD}\left(\frac{-\int_D^{\bar{\pi}} F_I\,d\pi}{1-F(D,I)}\right) \gtrless 0,\ \forall D \tag{12}$$

to prove the last claim.

The sign of the derivative of $\int_D^{\bar{\pi}} -F_I\,d\pi/(1-F(D))$ with respect to D depends on the sign of

$$F_I(D)(1-F(D)) + f(D)\int_D^{\bar{\pi}} -F_I\,d\pi.$$

Now, consider the MSP–case, for which monotony of $-F_I/f$ implies:

$$\begin{aligned}\frac{-F_I(\pi)}{f(\pi)} - \frac{-F_I(D)}{f(D)} &> 0 \quad \forall\pi > D \\ -F_I(\pi)f(D) + F_I(D)f(\pi) &> 0 \quad \forall\pi > D\end{aligned}$$

Integration yields:

$$f(D)\int_D^{\bar{\pi}} -F_I(\pi)\,d\pi + F_I(D)\int_D^{\bar{\pi}} f(\pi)\,d\pi > 0$$

The proof for the MPC–case is analog. □

Proof of Proposition 4

The claims about $x^*(\pi)$ follow immediately from the first order condition. To establish the slope of x rewrite the first order condition as $L_x \equiv a'f - a''(1-F)((\lambda-1)/\lambda) = 0$. It follows that $x' = -L_{x\pi}/L_{xx}$. Assumption 2 implies that

$$L_{xx} = a'''(1-F)((\lambda-1)/\lambda) - a''f < 0,$$

as is required by second order condition. Using $L_{x\pi} = a'f' + fa''((\lambda-1)/\lambda) - L_{xx}$ we obtain:

$$x' = 1 + \frac{a'f' + fa''((\lambda-1)/\lambda)}{-L_{xx}}$$

which is larger than one provided that $f'/f + (a''/a')((\lambda-1)/\lambda)$ is positive. Substitution of the second term using the first order condition yields: $f'/f + f/(1-F)$ which is positive by log–concavity of $1-F$. □

Proof of Corollary 1

Since $x(\pi)$ is monotone it can be inverted to obtain $\pi(x)$ with $\pi' = 1/x' < 1$. Substitute in the identity (7), rearrange and differentiate to obtain: $s'(x) = \pi' \cdot (1 - w' - a') + a'$, which simplifies by the incentive constraint (8) to $s'(x) = a'$ which is positive. From this we obtain $s''(x) = a'' \cdot (\pi' - 1)$ which is negative. □

Proof of Proposition 5

To ease notation let B denote the square bracketed integrant in program 3. The first order condition for the optimal choice of I is:

$$0 = Bf\frac{d\pi}{dI}\bigg|_{\underline{\pi}}^{\bar{\pi}} + \int_{\underline{\pi}}^{\bar{\pi}} Bf_I\, d\pi + \int_{\underline{\pi}}^{\bar{\pi}} (1-\lambda)(1-a')\frac{\partial}{\partial I}\frac{1-F}{f} f\, d\pi - \lambda \tag{13}$$

Partial integration of the second term yields:

$$\begin{aligned} \int_{\underline{\pi}}^{\bar{\pi}} Bf_I\, d\pi &= BF_I|_{\underline{\pi}}^{\bar{\pi}} - \int_{\underline{\pi}}^{\bar{\pi}} \frac{dB}{d\pi} F_I\, d\pi \\ &= BF_I|_{\underline{\pi}}^{\bar{\pi}} - \int_{\underline{\pi}}^{\bar{\pi}} \left(\frac{\partial B}{\partial \pi} + \frac{\partial B}{\partial x}x'\right) F_I\, d\pi \end{aligned} \tag{14}$$

Since $F(\underline{\pi}(I), I) = 0$ respectively $F(\bar{\pi}(I), I) = 1, \forall I$ we can substitute in (13) with $d\underline{\pi}/dI = -F_I(\underline{\pi}, I)/f(\underline{\pi}, I)$ and $d\bar{\pi}/dI = -F_I(\bar{\pi}, I)/f(\bar{\pi}, I)$. Hence the first term in 13 and in 14 cancel out.

Note that for $\lambda = 1$ equation (10) implies that $x = \pi$. From $x' = 1$ and $\partial B/\partial\pi = 1 - \partial B/\partial x$ follows $dB/d\pi = 1$. Therefore, (13) simplifies to $1 = -\int_{\underline{\pi}}^{\bar{\pi}} F_I\, d\pi$.

For $\lambda > 1$, first order conditions require $\partial B/\partial x = 0$ implying:

$$\frac{dB}{d\pi} = \frac{\partial B}{\partial \pi} = \lambda + (1-\lambda)(1-a')\frac{\partial}{\partial \pi}\left(\frac{1-F}{f}\right)$$

Hence (13) can be rewritten as

$$\begin{aligned} 0 &= -\lambda \int_{\underline{\pi}}^{\bar{\pi}} F_I\, d\pi \\ &\quad - \int_{\underline{\pi}}^{\bar{\pi}} (1-\lambda)(1-a')\frac{\partial}{\partial \pi}\left(\frac{1-F}{f}\right) F_I\, d\pi \\ &\quad + \int_{\underline{\pi}}^{\bar{\pi}} (1-\lambda)(1-a')\frac{\partial}{\partial I}\left(\frac{1-F}{f}\right) dF - \lambda \end{aligned}$$

Substitution of variable using $d\pi = dF/f$ and rearranging yields equation (11). Since $1 - \lambda < 0$ and $1 - a' > 0$, it remains to be shown that sign γ = sign $\frac{d}{d\pi}[-F_I/f]$ to prove the last claim. Rewrite γ as:

$$\gamma = \frac{1-F}{f^3}(f'F_I - f_I f) = \frac{F_I^2}{f^2}\frac{1-F}{f}\frac{\partial}{\partial \pi}\left(\frac{f}{F_I}\right)$$

The first two terms on the right hand side are positive and sign $\partial(f/F_I)/\partial\pi$ = sign $\partial(-F_I/f)/\partial\pi$. □

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