

Coordination Failure with Multiple-Source Lending,
the Cost of Protection Against a Powerful Lender

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Abstract

In this paper we analyze how a firm might protect quasi-rents in an environment of imperfect capital markets, where switching lenders is costly to the borrower, and contracts are incomplete. As switching costs make the firm vulnerable to ex-post exploitation, it may want to diversify lending in order to prevent opportunistic recontracting by the creditor. Multiple-source lending, however, suffers from coordination failure. An uncoordinated withdrawal of funds will force a financially distressed firm into bankruptcy even though it could have been rescued if lenders stayed firm. We show that the gains from preventing renegotiation do outweigh the cost of coordination failure if a single lender has sufficient bargaining power.

JEL: G33

1 Introduction

Financial theory has stressed an important advantage of short-term debt. It can be called in upon the arrival of unfavourable information about the firm's prospects even if this information is not contractible. This lowers ex ante financing cost in at least two ways. First, the firm will work hard to make bad news less likely and second, if the project goes awry, the creditor can prevent the firm from plundering the investment by pulling out in time. However, the flexibility of short-term claims is not without cost. If capital markets are not perfect, the withdrawal of a major lender can be troublesome even for a healthy firm. Due to indivisibilities and the sunk-cost nature of much investment the firm cannot simply sell part of its assets and scale down operations in response to a withdrawal of funds. Hence, it has to refund from new sources, which may be costly to find on short notice. Readily available credit such as trade credit or overdrafts from an alternative bank are expensive, and less informed lenders may take the pull out as bad news, which would raise financing cost even further. As switching to other sources of financing gets more costly, the old lender gains bargaining power over the firm's profit and may appropriate quasi-rents when refinancing becomes necessary (Sharpe (1990), Rajan (1992)). In order to mitigate the holdup problem the firm may want to raise funds from many lenders, thus strengthening its bargaining power at contract renewal.

This paper analyzes the costs and benefits of such a strategy. Suppose the firm has many lenders who are in the position to terminate their funding. This may happen because short-term loans are due or because the firm has violated some debt-covenants, or defaulted on payments such that long-term loans can be foreclosed. If the situation is bad enough, it will be obvious that the firm cannot be saved, and all lenders will take the chance and pull out. At the other extreme it may be clear that the firm's survival is not at risk and lenders will readily roll-over their loans to refinance the firm. But there may be circumstances in which the firm is doomed to fail if too many old lenders withdraw, while it could be saved if enough refinancing is provided in a timely manner. In such a situation each lender faces a choice between playing safe by pulling out and taking a risk by providing fresh money (by not foreclosing old loans).¹ Given that early credit terminations reduce the likelihood of success, lenders will be weary of the possibility to be left alone, and may resort to the very pre-emptive action which undermines the project.

¹It is assumed that the firm cannot fully collateralize all funds which are needed to keep it afloat. Hence we focus on risky debt and to the extent that loans are collateralized they are considered as being withdrawn.

Clearly, lenders could gain by negotiating with each other to achieve cooperation rather than acting independently and uncoordinated. And ultimately, when the firm files for bankruptcy, lenders will be forced to cooperate. German banks are known to have some success in coordination in an earlier stage by forming creditor pools for firms in financial distress (see Brunner and Krahnen (2000) for a detailed assessment). However, in this paper we rule out cooperation and focus on the risk of coordination failure. This appears justified for two reasons. First, once lenders coordinate their strategies the issue of bargaining power reemerges, and multiple-source lending might even be worse than single-source lending for the firm (Bolton and Scharfstein (1996)). So in order to analyze the drawbacks of multiple-source lending as a protection against ex-post exploitation of quasi-rents, we have to assume that lenders do not easily coordinate their strategies. Second, the assumption of non-cooperative behavior appears adequate for most healthy firms as well as for many firms in financial distress. Even if the firm defaults on some of its claims, it will still take time for lenders to set up a framework for negotiations. Often it is not in the interest of a lender, who obtains knowledge of financial difficulties, to share this information quickly with other lenders. By withholding information he may gain time to call in poorly collateralized loans or ask for additional collateral. Acting silently, he may improve his bargaining position vis-à-vis other lenders in later negotiations. Therefore, our analysis applies to pre-emptive action undermining the project in the run up to some private workout activity or a formal bankruptcy procedure.

The paper does not address other cost of multiple-source lending, which have been dealt with in the literature. As the number of lenders increases, the total cost of becoming informed and monitoring the firm may become prohibitive. In this case the firm faces a trade-off between ‘informed’ single-source lending and ‘uninformed’ multiple-source funding. Whereby inside debt, presumably provided by a bank, is ex-ante cheap but ex-post powerful, while arm’s-length debt is initially more expensive but powerless at a later stage. The issue has been analyzed in great detail by Rajan (1992), who compares a single informed lender, the bank, and many uninformed bondholders. Here we share with Rajan (1992) the assumption that dispersed debt will not be renegotiated, but we allow for many informed lenders. In a sense bargaining power can be diluted without incurring additional information cost.

Empirically, many firms diversify their bank lending, which does not necessarily rule out that they maintain a special relationship with one of them.² In this paper we

²For empirical evidence on lending patterns and the importance of relationship lending see

abstract from differences among lenders and consider only the admittedly extreme cases of a large number of lenders and a single lender. We assume that all banks have equal capacity to read the firm's information at no additional cost and show that there is a disadvantage of relying on many lenders even if these are no less informed than a single lender. Hence, coordination failure may also provide a justification for giving one lender a leading role.

Bolton and Scharfstein (1996) discuss the optimal number of creditors from the perspective of ex-post bargaining problems. In order to induce the firm to serve its debt, lenders retain the right to seize assets in case of default. A large number of lenders deters strategic default because the firm would have to go through multi-lateral negotiations to regain control of the assets. This would leave the firm with a lower payoff than with two-party bargaining. However, in the case of liquidity default, the firm lacks funds to buy back the assets and lenders would have to find an outside investor. The outside investor will anticipate that the price to be paid is larger with many lenders. If he has to incur some cost up front in order to assess the value of the assets, he might choose not to enter the negotiations at all. Hence, having many lenders may be a disadvantage in the case of liquidity default.

From the more technical side, the paper is related to the fast growing literature on global games. With strategic complementarities these often have a unique (dominance solvable) equilibrium which greatly simplifies the analysis of coordination failure.³ We will further discuss this relation when analyzing the coordination problem.

The next section describes the model. In section 3 we analyze the interaction between multiple lenders and assess the cost of coordination failure. In section 4 we compare the results to the case of a single lender who renegotiates the loan. We briefly discuss bargaining power in section 5 and conclude in section 6. Most proofs have been relegated to the appendix.

2 The Framework

A firm has a fixed-scale project, which takes two periods to mature. In both periods it faces non-contractible cost of carrying the project one step ahead, denoted V_1 ,

Petersen & Rajan (1995) (USA), Hoshi & Kashyap & Scharfstein (1990) (Japan), and Elsas & Krahenen (1999) (Germany), Farinha & Santos (2000) (Portugal).

³See Morris and Shin (2001) for an overview stressing methodological issues and providing several applications, Morris and Shin (1999) on the pricing of debt, and Goldstein and Pauzner (2000) on bank runs.

respectively V .⁴ There are several possible interpretations for these costs. They may measure the disutility of effort required to manage the project successfully. Alternatively, they may indicate the value of assets which the firm can steal once it decides to abandon the project, the benefits of ‘consumption on the job’, etc. Since little depends on the exact nature of the incentive problem, we will usually refer to V_1 and V as ‘effort’. If the firm fails to provide ‘effort’ in any one period, the project becomes worthless. If continued through both periods, it yields a return θ . Ex ante, θ is a random variable with uniform distribution on $\Theta = [\underline{\theta}, \bar{\theta}]$, $\bar{\theta} - \underline{\theta} = 1$.

Lenders hold debt-like claims. If they call in their loans (foreclose) at the end of the first period, they receive K . To simplify the exposition we assume that K does not depend on how many lenders withdraw. The firm may have enough assets at this stage, or it may borrow from uninformed new lenders whatever is needed to satisfy old claims, even if it will default later on. If lenders decide to roll over the loan their payoff is $D > K$, provided the firm has contributed V_1 and does not fail in the second period. If the firm should fail the project would be worthless and the payoff is zero. Multiple lenders decide non-cooperatively about foreclosure or extension, and $T \in [0, 1]$ denotes the fraction of loans which are terminated. We assume that multiple lenders do not bargain over D . This is a reasonable assumption as a minor lender rarely has the bargaining power to force the firm into loan renegotiation. A single lender, however might also renegotiate the loan and settle for a new claim D^* .

Finally, knowing the expected return θ , the firm decides whether to plunder the project and withhold effort, gaining V , or to strive for a successful completion. In the latter case it has to refinance from new lenders (or forego economies of scale). The total cost of an outflow of funds K is W , hence, the firm’s payoff from continuation is $\pi_S = \theta - D - T(W - D)$, where the last term indicates the switching cost. It is assumed that $W > D$ so that switching lenders is to the disadvantage of the firm. Recall that without switching cost of some kind no hold-up problem would exist, and thus diluting the creditors’ power would not be necessary. As discussed in the introduction, such cost may reflect that new lenders require a mark-up for refinancing a firm in financial distress, or that the firm incurs cost to find new financiers. It may also temporarily resort to more expensive sources of financing, such as trade credit or overdrafts, usually being employed for cash-management but not for investment funding. As this paper is about the implications of switching cost and not about their nature, we make the simplifying assumption that switching cost are proportional to the amount of funds withdrawn. However, our qualitative results go through, provided that total switching cost are increasing in the termination rate

⁴We state all cost and payoffs in expected terminal wealth, so that discounting is not an issue.

T and nonincreasing in θ .

The basic structure of the model is summarized by the following timing of moves:

1. Given financial claims (K, D) , the firm provides or withholds V_1 (observable but not verifiable by lenders),
2. lenders terminate loans (T) or refinance $(1 - T)$; a single lender might also renegotiate the loan and settle for a new claim D^* ,
3. knowing θ , the firm may either plunder the project or forego V , refinance at additional cost $W - D$ and carry on.

In the last stage of the game, when the firm considers whether to aim for a successful completion of the project or to fail, it will take into account its obligations to financiers. Hence it fails if:

$$V \geq \theta - D - T \cdot (W - D) \tag{1}$$

Even if all credit is extended ($T = 0$) the firm will default provided that $\theta \leq \theta_0 = V + D$, while it will carry on even if all credit is withdrawn for $\theta > \theta_1 = V + W$. It is the intermediate range $\theta \in (\theta_0, \theta_1]$ which is critical. Here the firm's success depends on the action of its lenders.

In the second stage lenders decide whether to extend credit or to terminate funding. As to their information we consider two variants. In the first, the perfect information case, lenders know θ before they decide whether to terminate or to refinance. In the second, each lender receives a noisy signal $x \in X$ revealing some information about θ . As in Morris & Shin (1998) noisy information enables us to avoid the multiplicity of equilibria in the coordination game.⁵ Our focus is however on multiple-source lending as an instrument to dilute lenders' power, not on signal quality, and we therefore emphasize the limiting case of perfect information. Following Morris & Shin (1998) we assume that the signals are independent and uniformly distributed with:

$$F(x|\theta) = \begin{cases} 0 & ; x < \theta - \epsilon \\ \frac{1}{2\epsilon}(x - \theta + \epsilon) & ; \theta - \epsilon \leq x \leq \theta + \epsilon \\ 1 & ; \theta + \epsilon < x. \end{cases} \tag{2}$$

The noise ϵ is small

$$0 < 2\epsilon < \min\{\theta_0 - \underline{\theta}, \bar{\theta} - \theta_1\}. \tag{3}$$

⁵While the effect is similar, the noisy information approach appears more natural than using ad hoc refinement criteria such as 'trembling hand perfectness'.

As will become clear below, condition (3) allows us to ignore signals for realizations of θ close to $\underline{\theta}$ and $\bar{\theta}$. Eventually, we are most interested in the limiting case of $\epsilon \rightarrow 0$.

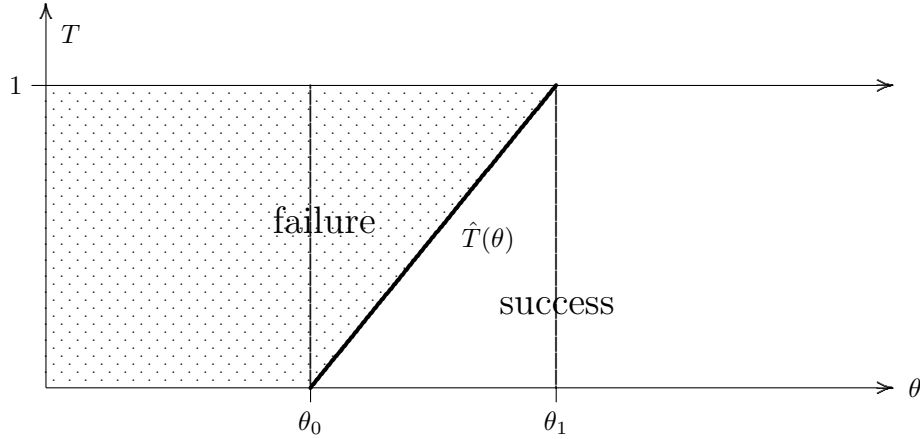
3 Many lenders

With many of them, lenders do anticipate how the firm will respond to the aggregate strategy of all lenders, but they do not play cooperatively against the reaction function of the firm. Even though moving first as a group, any single lender takes the other lenders' strategies and the subsequent behavior of the firm as given. With respect to the latter a unique threshold $\hat{T} = (\theta - V - D)/(W - D)$ can be derived from condition (1), such that the firm will serve the debt if and only if the fraction of loans withdrawn is kept below this threshold, i.e. $T < \hat{T}(\theta)$. The schedule is illustrated in figure 1.

Anticipating the firm's reaction to the aggregate rate of withdrawal, a lender extends the loan if the expected returns from rolling-over, R , are larger than what he gains by foreclosing, K . With strategic interaction between lending decisions, rational lenders must give a thought to how their fellow lenders will act, and, therefore, how they themselves should behave. We will consider two variants with respect to the lenders' capabilities to analyze their fellow lenders' perception of the problem. According to the first, all lenders are able to solve not only their own but also each others' optimization problem. In other words, a lender can predict the optimal behavior of any other lender for every possible assumption this lender may hold about the behavior of lenders as a group. For this variant the commonly used Nash-equilibrium is the appropriate concept of solution. However, we are also interested in what happens when the lenders' analytical abilities are more limited. Therefore, we consider the case when lenders can only identify strictly dominated strategies, which appears much less demanding than positively deriving optimal strategies. With this assumption we can narrow down the set of possible outcomes through the iterative elimination of dominated strategies (Bernheim (1984)).⁶ It is well known that the set of strategies which are rationalizable in this sense includes the set of Nash-equilibrium strategies. By looking for Nash-equilibria and rationalizable equilibria, we account for different possibilities regarding the intellectual abilities of lenders to solve problems of strategic interaction.

⁶In both variants the structure of the game is common knowledge, i.e. each lender knows the abilities of all other lenders and knows that the other know that he knows etc....

Figure 1: Critical Rate of Termination



Perfect Information

For a start, consider the benchmark case of perfect information in which lenders observe θ . For $\theta \leq \theta_0$ a lender knows that failure is certain no matter what the other lenders do. Hence the best he can do is withdraw. And since the firm will never cheat for $\theta > \theta_1$ refinancing is dominant in these states. Hence, for $\theta \leq \theta_0$ and $\theta > \theta_1$ there exist unique equilibria in dominant strategies, terminate and refinance, respectively. For the intermediate range $\theta \in (\theta_0, \theta_1]$, however, neither of the two options is dominated. Hence, both strategies can be rationalized by plausible assumptions about the behavior of the other lenders. Moreover, there even exist multiple Nash-equilibria. If a lender expects all other lenders to foreclose, then $T = 1 > \hat{T}$. The firm would fail and the best response is to foreclose also. Hence, all lenders expecting all others to terminate is a self-fulfilling equilibrium, but so is $T = 0 < \hat{T}$. If everyone else refinances, the firm will carry on and it is again rational to follow suit.

Arguably multiple equilibria can be taken as evidence of a coordination problem. But then it is difficult to go beyond this statement. With multiple equilibria, lenders cannot use the model to assign probabilities to possible outcomes — they would face true uncertainty in the sense of Knight (1921). Unfortunately, the analysis of rational decision-making without subjective probabilities is still in its infancy.⁷ Without a generally accepted methodology to assess the severity of the coordination problem, it is impossible to compare the cost and benefits of multiple-source lending with single-

⁷See Schmeidler (1989) and Wakker (1989) for such an attempt to reformulate utility theory.

source lending. Even worse, multiple equilibria undermine the very foundations of equilibrium analysis. With a unique Nash–equilibrium any theory recommending the choice of a non–equilibrium strategy would be self–defeating if adopted by everyone. This argument fails for multiple equilibria, and there is no reason why players should be concerned with equilibrium strategies at all if these are not unique.

One possible solution, is to use a priori reasoning to select a particular equilibrium as more plausible than others. A fairly common approach is to rule out all Pareto–dominated equilibria. This, however, stretches the requirements of common knowledge even further. Not only, must all agents be able to solve the optimization problem of all other agents, they must also know that all players use Pareto–dominance as the selection criterion when confronted with multiple equilibria. In our case this criterion solves the dilemma. All lenders strictly prefer the refinancing equilibrium over any other (possibly mixed strategy) equilibrium because of $D > K$. Hence, Pareto–dominance yields refinancing ($T = 0$) as the unique equilibrium in the critical range. Thus, the coordination problem completely disappears and the firm could dilute creditors’ bargaining power at zero cost through diversification of lending.

In the case of perfect information, the existence of the coordination problem depends entirely on the degree of sophistication which we are prepared to assume for the lenders.

Imperfect Information

When information is imperfect, lenders have to base their decision on the noisy signal x . Let S denote the set of all functions $t : X \rightarrow [0, 1]$. The strategy of a lender i consists of a rule $t_i \in S$ indicating for each possible observation x the probability of foreclosure. Aggregating over all lenders we obtain the fraction of loans which are scheduled for withdrawal for any x . This is also an element of S and will be denoted $t(x)$. As $t(x)$ summarizes the strategic decisions of the lenders, it is all we need to know about the strategy profile. Since signals are uniformly distributed, it is easy to derive the aggregate behavior implied by any strategy profile t . The fraction of loans which will be terminated when the true state is θ is given by $T(\theta, t) = \frac{1}{2\epsilon} \int_{\theta-\epsilon}^{\theta+\epsilon} t(x) dx$. Define $x_0 = \theta_0 - \epsilon$ and $x_1 = \theta_1 + \epsilon$. For observations $x \leq x_0$ and $x > x_1$ a lender knows that $\theta \leq \theta_0$ (certain failure) respectively $\theta > \theta_1$ (certain success). As in the full information case, lenders have dominant strategies for extreme observations, $t(x) = 1$ and $t(x) = 0$, respectively. It is only for intermediate values $x \in (x_0, x_1]$ that the functions t_i and t can take any values in $[0, 1]$.

In the following we characterize the equilibrium for arbitrary strategies $t \in S$. However, in order to develop some intuition for the main result in a technically more accessible way, we assume that lenders employ switching strategies $t_{\hat{x}}$, such that credit is rolled over if and only if x is larger than some threshold $\hat{x} \in [x_0, x_1]$.

$$t_{\hat{x}}(x) = \begin{cases} 1 & \text{for } x \leq \hat{x} \\ 0 & \text{for } x > \hat{x} \end{cases}$$

With this strategy the rate of terminations T equals the fraction of lenders observing $x \leq \hat{x}$, given that θ is the true state, hence from (2) we obtain

$$T(\theta, t_{\hat{x}}) = F(\hat{x}|\theta) = \begin{cases} 1 & \text{for } \theta \leq \hat{x} - \epsilon \\ \frac{1}{2\epsilon}(\hat{x} - \theta + \epsilon) & \text{else} \\ 0 & \text{for } \hat{x} + \epsilon \leq \theta. \end{cases}$$

Note that $T(\theta, t_{\hat{x}})$ is decreasing in θ , thus $T(\theta, t_{\hat{x}}) = \hat{T}(\theta)$ defines a unique cut-off point

$$\hat{\theta}(\hat{x}) = \frac{\epsilon(W + D + 2V) + (W - D)\hat{x}}{W - D + 2\epsilon}, \quad (4)$$

such that the firm will fail whenever $\theta \leq \hat{\theta}(\hat{x})$. Note that $\hat{\theta}$ is increasing in \hat{x} . As the threshold required by the lenders for extending credit is raised, the range of returns for which the firm defaults is increased.

Now, consider an individual lender having observed x . Given a strategy profile $t_{\hat{x}}$, the expected profit from rolling over his loan is

$$R(x, t_{\hat{x}}) = (1 - p(\hat{\theta}|x))D, \quad (5)$$

where

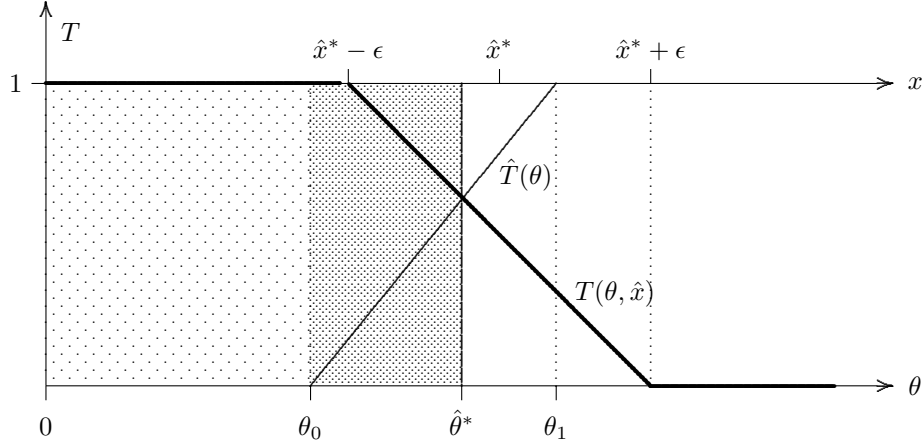
$$p(\theta|x) = \begin{cases} 0 & \text{for } \theta \leq x - \epsilon \\ \frac{1}{2\epsilon}(\theta - x + \epsilon) & \text{else} \\ 1 & \text{for } x + \epsilon \leq \theta \end{cases} \quad (6)$$

denotes the conditional probability distribution of θ . As in the full information case, we have strategic complementarity among the decisions of the lenders. A creditor who contemplates on whether to extend his loan would welcome if other lender were to lower their threshold and roll over credit in more states, because this would make default less likely.

Since $\partial R/\partial x > 0$, there exists a unique solution to $R(x, t_{\hat{x}}) = K$, yielding the threshold \hat{x} , at which the individual lender will switch from termination to refinancing given the firm's failure point $\hat{\theta}$:

$$\hat{x}(\hat{\theta}) = \hat{\theta} + \epsilon \left(\frac{2K}{D} - 1 \right). \quad (7)$$

Figure 2: Equilibrium with multiple lenders



If all other lender increase their threshold for refinancing, the firm's threshold for failure $\hat{\theta}$ is raised which increases the optimal \hat{x} . Solving equations (4) and (7) yields the unique Nash-equilibrium among the class of switching strategies which is illustrated in figure 2. It is obtained by blending the threshold \hat{T} from figure 1 with the proportion of lenders withdrawing their loans T . Having opposite slope in the critical region, both graphs intersect once at the most. An increase of the switching point \hat{x} shifts the T schedule to the right. An equilibrium is reached if the expected payoff from refinancing at the switching point, $R(\hat{x}, t_{\hat{x}})$, equals the return from foreclosing. Since the lenders' payoffs exhibit strategic complementarity, the former is strictly increasing in \hat{x} . Hence there can only be one such point.

The next proposition establishes that this solution is also the unique Nash-equilibrium if arbitrary strategies are allowed for.

Proposition 1 *With multiple lenders there exists a unique Nash-equilibrium in pure strategies with every lender switching from foreclosure to refinancing at*

$$\hat{x}^* = \hat{\theta}^* + \epsilon \left(\frac{2K - D}{D} \right), \quad (8)$$

and the firm's failing point is given as

$$\hat{\theta}^* = \theta_0 + \frac{K(W - D)}{D}. \quad (9)$$

The uniqueness of the Nash-equilibrium in the imperfect information case stands in marked contrast to the ambiguity of the full information case. Due to strategic

complementarity, the result can be further strengthened. $t_{\hat{x}^*}$ is also the unique strategy profile surviving the iterative elimination of dominated strategies, i.e.

Corollary 1 $\{\hat{\theta}^*, \hat{x}^*\}$ is the unique rationalizable equilibrium.

Ironically, perfect information about the fundamentals leads to profound uncertainty. Even if it is common knowledge that players are able to solve each others' optimization problems, there is little basis to guide rational choice. However, if information about the fundamentals is slightly disturbed, then all indeterminacy in the players' strategies is removed. Even if players are only able to identify dominated strategies, we obtain a unique prediction for rational behavior.

As Morris & Shin (2001) show these results on the coordination problem can be considerably generalized and strengthened. In particular they propose a simple rule of thumb-procedure which is far less demanding than the elimination of dominated strategies, and nevertheless yields the unique equilibrium strategy. It is sufficient if players can estimate (i) θ from the signal, (ii) postulate that the proportion of players withdrawing, T , is distributed uniformly on the unit interval, and (iii) take the optimal action. This simple procedure works in a much wider class of symmetric binary action global games.⁸

It is somewhat surprising that the firm's equilibrium failure point $\hat{\theta}^*$ does not depend on the precision of the information ϵ . This is, however due to the particular functional forms such as the linearity of the threshold function \hat{T} , and the uniform distribution of the signal. In general, the failure point will depend on the informativeness of the signal, although failure may become less, as well as more, likely as precision improves (Heinemann & Illing (1999) and Morris & Shin (1999)).

It is worthwhile emphasizing that, even as information gets ever more precise ($\epsilon \rightarrow 0$), the unique rationalizable equilibrium in the imperfect information case does not approach the unique Pareto-dominating Nash-equilibrium of the full-information case. We therefore obtain a non-trivial and tractable coordination problem even in the limit of the perfect information case. The difference between the ranges of coordination failure in the two cases is illustrated in figure 2 by the more darkly shaded area to the left of $\hat{\theta}^*$. This casts strong doubts on the usefulness of Pareto-dominance as a criterion for equilibrium selection in coordination problems.

⁸For further discussion on the uniqueness of the equilibrium in the wider class of symmetric binary action global games, the role of supermodularity, and the intuition why this rule of thumb yields equilibrium strategies see Morris & Shin (2001)

The Cost of Inability to Commit

In the remainder of this section we want to clarify the cost of multiple–source lending by comparison to a single lender who can commit not to renegotiate the conditions for refinancing. One may think of a bank which has developed a reputation as a strict but fair lender. In other words we rule out the problem of ex post exploitation, which motivated our research of diversified lending.

For the lender the return from contract termination is the same as with many lenders, but when calculating the return to extending the credit, he will take into account that the firm will respond to his decision. When he extends the credit, the firm will fail up to θ_0 , and when he terminates the loan, the firm fails up to θ_1 .⁹ Hence, a committed lender switches at

$$K = (1 - p(\theta_0|x))D,$$

which yields:

$$\hat{x}(\theta_0) = \theta_0 + \epsilon \left(\frac{2K}{D} - 1 \right). \quad (10)$$

The probability that the lender observes an $x \leq \hat{x}(\theta_0)$ and terminates the loan is again given by $T(\theta, \hat{x}(\theta_0)) = F(\hat{x}(\theta_0)|\theta)$. With multiple lenders the firm fails with certainty once expected returns fall below θ^* and succeeds with certainty otherwise. If a single committed lender observes a signal $x \leq \hat{x}(\theta_0)$, he withdraws and the firm will fail. If the signal is above that threshold, he will extend the loan and the firm will succeed, provided that $\theta \geq \theta_0$. Hence, the outcome is uncertain for $\theta \in [\theta_0, \hat{x}(\theta_0) + \epsilon]$.

Proposition 2 *For both sides expected profits are higher with a single committed lender than with multiple lenders. This Pareto–gain is achieved without any other compensating changes in the contract (K or D).*

Conditional on the credit being extended, single and multiple lenders bear the same expected loss due to firm failure. But a single committed lender can reap the gains from rolling over the loan more often, because he extends credit in more states

⁹In principle, a single lender might want to terminate only a fraction T of his loan, in which case his expected profits would be:

$$\Pi^S = TK + (1 - T)(1 - p(\hat{\theta}(T)|x))D.$$

However, since $\partial^2 \Pi^S / \partial T^2 > 0$ an interior solution would never be optimal.

of nature than many uncoordinated lenders, $\hat{x}(\theta_0) > \hat{x}(\hat{\theta}^*)$. Hence, the expected profits of a single committed lender are higher. The firm also gains from lowering the threshold for extending credit.

These gains from shifting the $T(\theta, \hat{x})$ -schedule to the left do not depend on the signal's precision. They are obtained even in the limiting case of $\epsilon \rightarrow 0$. When the signal is unreliable ($\epsilon > 0$), however, there is an additional gain for the firm in good states. With probability $T(\theta, \hat{x})$ a single lender receives a bad signal and denies credit. The firm subsequently fails and obtains only V . With multiple lenders, $T(\theta, \hat{x})$ is the fraction of lenders who observe bad signals and pull out. This harms profits even if it does not prevent the firm from succeeding. On the new loans the firm earns only $\theta - W$, which is less than its opportunity cost V by definition of θ_1 . It carries on only because the returns from extended old loans are large enough to 'subsidize' expensive fresh loans. Therefore, when the signal is noisy, expected profits of the firm would be higher with a committed single lender even if there would be no change in the threshold for extending loans.

4 Single Lender with Recontracting

So far we have analyzed coordination failure as a possible disadvantage of multiple-source lending. This raises the question as to why a firm does not simply lend from one single source. As we have just seen, if such a lender could commit not to renegotiate the loan, both sides would in fact be always better off. However, in most monopolistic lending relationships it will be difficult to establish the credibility of such a promise. Thus, the more relevant scenario appears to be that a lender cannot commit *ex ante* on the conditions for the refinancing. In the following we derive conditions for which single-source lending without commitment is inferior to multiple-source lending.

After receiving his information about the firm's profitability, the lender may renegotiate the loan in an attempt to appropriate the firm's profits. In doing so he is constrained by the firm's outside option to refinance at cost W and by the need to provide incentives for performance. Within these limits the bargaining power may vary. Since Rajan (1992) provides a comprehensive analysis of the issue, we adopt the simplifying assumption that the lender enjoys all the bargaining power e.g. by making a single take-it-or-leave-it offer. If the lender refinances, then he charges D^* defined as

$$D^* = \arg \max_{D \leq W} (1 - p(\theta_0|x))D,$$

and he will refinance rather than terminate, provided that $(1 - p(\theta_0|x))D^* > K$.

Assuming that ϵ is small enough so that $\epsilon < K/2$, we can characterize the equilibrium as follows:

Proposition 3 *A monopolistic lender enjoying full bargaining power switches to refinancing at*

$$\hat{x}^R = K + V + \epsilon \quad (11)$$

and charges

$$D^* = \begin{cases} x - \epsilon - V & \text{for } \hat{x}^R < x < W + V + \epsilon \\ W & \text{for } W + V + \epsilon \leq x \end{cases}$$

for the new loan. The firm will always fail for $\theta \leq K + V$ and never fail for $\theta > K + V + 2\epsilon$.

A monopolistic lender extends credit even more often than a lender who is committed to charge D . This is due to the fact that he will soften the loan in some lower range of observations. However, as the signal gets better he will tighten loan conditions. Provided the signal is precise enough, he will do so avoiding any risk of default. In other words, if he refinances, then he will set D^* as high as possible, but low enough to make sure that the firm will not default.

Obviously, with full bargaining power, the lender's expected payoff conditional on the firm's performance in the first period cannot be smaller with the additional option to renegotiate. With respect to the firm's profit we look at the limiting case of $\epsilon \rightarrow 0$.

Proposition 4 *In the limit of vanishing noise the firm is strictly better off with multiple lenders than with a single lender who enjoys full bargaining power.*

In the limit the firm will be just indifferent between failing and carrying on for $\theta \in [V + K, \theta_1]$. Thus exploited by a monopolistic lender, the firm's payoff is

$$\pi_F^R = \int_{\underline{\theta}}^{\theta_1} V d\theta + \int_{\theta_1}^{\bar{\theta}} (\theta - W) d\theta.$$

While it obtains

$$\pi_F = \int_{\underline{\theta}}^{\hat{\theta}^*} V d\theta + \int_{\hat{\theta}^*}^{\bar{\theta}} (\theta - D) d\theta.$$

with multiple lenders. From $\hat{\theta}^* < \theta_1$ and $V < (\theta - D)$, $\forall \theta > \theta_0$ we conclude that $\pi_F > \pi_F^R$. At the refinancing stage, the firm is strictly worse off with a single lender who enjoys full bargaining power than it is with multiple lenders, in spite of the coordination problem. This in turn jeopardizes the firm's incentive to provide effort in the early phase of the project. There exists a range of parameters $V_1 \in [\pi_F, \pi_F^R]$, for which the firm would never provide effort at the first stage if financed by a single lender. Hence, it would be in the interest of both sides to bring in additional lenders in order to limit the bargaining power of the creditor, even though this comes at the cost of possible coordination failure.

5 Shifting Bargaining Power to the Firm

The focus of this paper is on how the firm might protect itself against ex-post exploitation by opportunistic creditors. This issue comes out sharpest when the creditor has all the bargaining power at the refinancing stage. In this extreme form, however, the assumption is difficult to justify, and we therefore briefly discuss what happens if we shift bargaining power to the firm.¹⁰ Obviously, weakening the lender will increase π_F^R thus decreasing the range of parameters for which it is worthwhile to incur the cost of coordination failure. At first glance this would make single-source lending more attractive. However, given our assumptions the firm can push the lender to a low reservation utility by threatening to withdraw and plunder the project. As the firm's bargaining power increases, it becomes ever more difficult to compensate the lender for his initial contribution to the project and the problem is reversed. It is beyond the scope of this paper to review the large literature on mechanisms that protect the creditor against opportunistic recontracting on part of the firm, but it is worthwhile noting that diversified lending may again be instrumental. The issue has been analyzed in the context of banking by Diamond and Rajan (2000). Suppose the firm (bank), in an attempt to exploit its lenders (depositors), offers a low payment and threatens to let the project down if the offer is not accepted. A single lender may accept such an offer if killing the project by early withdrawal of funds is even worse. This is different with a large number of lenders. Assuming that others will accept the offer, a single lender will become reluctant to do so. Expecting that the firm will survive, he may obtain a higher payoff by quickly

¹⁰It is beyond the scope of this paper to explicitly analyze the bargaining problem — which, for $\epsilon > 0$, is one with asymmetric information. However, any solution to contract renegotiation will only depend on parameters K , V , W and, possibly, a parameter for bargaining power. A role for the contract variable D can only be established if renegotiation is prevented.

foreclosing the loan rather than becoming soft. Again, a massive pullout forces the firm into default. In banking theory this happens if the liquidation value of assets is smaller than the value of outstanding debts and if there is no lender of last resort. Expecting default, it becomes ever more important to be the first in line to withdraw the loan. In equilibrium, the lender's uncoordinated attempt to cash in their loans, the bank run, will make the firm's threat to fail immediately become reality. Since this leaves both sides worse off, the firm's ability to renege on promised payments is curbed. In effect, failure to coordinate serves as a credible commitment not to renegotiate. Again, this commitment is costly if there are some states in which the firm is forced into bankruptcy if it cannot obtain concessions from its creditors.

6 Concluding Remarks

A lender who learns that all other lenders had called in their loans, just after he rolled over his credit to a firm may be worried for two reasons. First, the withdrawals suggest that his judgment of the firm's prospects was too optimistic. Second, as many lenders retreat, the chances for the firm's survival may be hampered even further, when capital markets are imperfect. By terminating a loan a creditor adds to the difficulties of the debtor, thus imposing a negative externality on the remaining creditors. An uncoordinated attempt to pre-empt default may lead to a downfall of the firm which could have been prevented if all lenders had stayed firm. Hence, with multiple-source financing a firm may be too often forced into bankruptcy.

If lenders are perfectly informed, the nature of the coordination problem largely depends on the assumption about the lenders' abilities to analyze and solve strategic games. However, if there is a minimal amount of noise in the lenders' assessment of the situation, then all the indeterminacy of optimal behavior is removed and coordination failure will occur for any rationalizable strategy. This results in excessive bankruptcies compared to a single lender, who accounts for the impact of credit extension on the firm's viability and refinance more often.

These cost of have to be balanced off against the advantages of diversified lending, i.e. the reduced bargaining power of creditors. When capital markets are imperfect and switching lenders is costly, a single lender might recontract the original contract in an attempt to appropriate quasi-rents. If he cannot commit to abstain from renegotiation and has sufficient bargaining power, then the firm is better off with multiple-source lending, in spite of the cost of coordination failure.

Since a powerful lender appropriates quasi-rents when the firm's prospects are

bright, while coordination failure happens if expected returns are sufficiently low, one might obtain the best of both worlds, with a multitude of lenders backed up by institutional arrangements facilitating quick coordination on the onset of financial difficulties. German banks, for example, have a practice of forming creditor pools for firms in financial distress (see Brunner and Krahnert (2000) for details). In contrast to a syndicated loan, the lenders have little communication as long as there is no concern about possible default. Upon the arrival of bad news, however, they often succeed in preventing uncoordinated withdrawal by pooling collateral and jointly developing a work out plan.

Appendix

Proof of Proposition 1

The following proof is adapted from Morris & Shin (1998). We start with a last result concerning switching strategies. Consider the expected returns to refinancing evaluated at an arbitrary common switching point \hat{x}

$$R(\hat{x}, t_{\hat{x}}) = \left(1 - \frac{1}{2\epsilon}(\hat{\theta}(\hat{x}) - \hat{x} + \epsilon)\right) D.$$

From (4) one obtains

$$d\hat{\theta}/d\hat{x} = (W - D)/(W - D + 2\epsilon) \leq 1$$

for the critical range. Hence, $\hat{\theta}(\hat{x}) - \hat{x}$ is strictly decreasing in \hat{x} , which makes $R(\hat{x}, t_{\hat{x}})$ strictly increasing in \hat{x} . In other words, as the signal for which the lenders switch to refinancing gets better, the payoff to refinancing conditional on receiving this signal gets higher. Note, that such a feature would ensure that the switching strategy \hat{x}^* solving $R(\hat{x}, t_{\hat{x}}) = K$ is unique, even if equations (4) and (7) were non-linear.

Now we turn to arbitrary strategy profiles t . Let $\hat{\Theta}(t)$ denote the set of states for which the firm fails:

$$\hat{\Theta}(t) = \{\theta | V + D \geq \theta - \frac{W - D}{2\epsilon} \int_{\theta - \epsilon}^{\theta + \epsilon} t(x) dx\},$$

and let $P(\hat{\Theta}(t)|x)$ be the probability of failure given a strategy t conditional on x :

$$P(\hat{\Theta}(t)|x) = \frac{\text{Prob}\{\theta | \theta \in \hat{\Theta}(t) \cap [x - \epsilon, x + \epsilon]\}}{\text{Prob}\{\theta | \theta \in [x - \epsilon, x + \epsilon]\}},$$

then the expected returns to refinancing are

$$R(x, t) = (1 - P(\hat{\Theta}(t)|x))D.$$

We define the following partial ordering of strategy profiles. For any $\underline{t}, \bar{t} \in S$, if $\underline{t}(x) \leq \bar{t}(x)$ almost everywhere then $\underline{t} \leq \bar{t}$. A strategy (profile) \bar{t} is larger than another strategy \underline{t} if lenders foreclose weakly more often. From the definition of $\hat{\Theta}$ it is obvious that $\hat{\Theta}(\underline{t}) \subseteq \hat{\Theta}(\bar{t})$, implying $P(\hat{\Theta}(\underline{t})|x) \leq P(\hat{\Theta}(\bar{t})|x)$, hence

$$\underline{t} \leq \bar{t} \implies R(x, \underline{t}) \geq R(x, \bar{t}) \tag{12}$$

The expected returns to extending credit is larger if other lenders refinance more often.

Now consider an equilibrium profile \tilde{t} . We show that every \tilde{t} must be the switching profile $\hat{t}_{\hat{x}^*}$. Define

$$\underline{x} = \inf\{x | \tilde{t}(x) < 1\} \quad \text{and} \quad \bar{x} = \sup\{x | \tilde{t}(x) > 0\}$$

Since $t(x) = 1$ for $x < x_0$ and $t(x) = 0$ for $x > x_1$ we know that $\underline{x} \leq \inf\{x | 0 < \tilde{t}(x) < 1\} \leq \sup\{x | 0 < \tilde{t}(x) < 1\} \leq \bar{x}$, hence

$$\underline{x} \leq \bar{x} \tag{13}$$

If some lenders refinance in equilibrium for a given signal x , then the returns to refinancing are at least as large as returns to foreclosure. By continuity this is also true at \underline{x} , hence $R(\underline{x}, \tilde{t}) \geq K$. Now compare this with the payoff of a switching strategy $R(\hat{x}, \hat{t}_{\hat{x}})$. Since $\hat{t}_{\hat{x}}(x) \leq \tilde{t}(x)$ it follows from (12) that

$$R(\hat{x}, \hat{t}_{\hat{x}}) \geq R(\underline{x}, \tilde{t}) \geq K$$

Since $R(\hat{x}, \hat{t}_{\hat{x}})$ is increasing in \hat{x} , and \hat{x}^* is the unique solution to $R(\hat{x}, \hat{t}_{\hat{x}}) = K$, we conclude that $\hat{x}^* \leq \underline{x}$. A symmetric argument shows that $\bar{x} \leq \hat{x}^*$, thus we have

$$\bar{x} \leq \hat{x}^* \leq \underline{x}.$$

Combining this with (13) we obtain

$$\underline{x} = \hat{x}^* = \bar{x}.$$

Hence, the equilibrium profile equals the switching profile $\hat{t}_{\hat{x}^*}$ almost everywhere. \square

Proof of Corollary 1

The claim can be proved by iteratively discharging dominated strategies (see Morris & Shin (1999) or Heinemann & Illing (1999) for this approach). This, however, is not quite necessary in view of a powerful theorem (Nr 5) by Milgrom & Roberts (1990) on equilibria in supermodular games.

Let $\mathcal{R} : S \times S \rightarrow \mathbb{R}$ denote a lender's expected profit from refinancing as it depends on his own strategy t_i and the strategy profile t of the other lenders:

$$\mathcal{R}(t_i, t) = \int_X (1 - t_i(x)) R(x, t(x)) dx$$

In order to use the theorem mentioned above, we have to show that the game played among the lenders is a 'supermodular' game. It is easy to verify that the

more technical conditions, such as S being a complete lattice (with respect to the ordering defined in the proof of proposition 1) and \mathcal{R} being order continuous in t_i and t , are fulfilled. The crucial point is that \mathcal{R} has increasing differences in t_i and t , i.e. that $\mathcal{R}(\bar{t}_i, t) - \mathcal{R}(\underline{t}_i, t)$ is nondecreasing in t .

$$\mathcal{R}(\bar{t}_i, t) - \mathcal{R}(\underline{t}_i, t) = \int_{x_0}^{x_1} [\underline{t}_i(x) - \bar{t}_i(x)]R(x, t(x)) dx$$

Since $\underline{t}_i(x) - \bar{t}_i(x)$ is nonpositive and $R(x, t(x))$ is nonincreasing in t by (12), the integral on the right hand side is nondecreasing in t . This establishes strategic complementarity for general strategy profiles. Given that the game is supermodular, we know from theorem 5 (Milgrom & Roberts (1990)) that there exist smallest and largest serially undominated strategies and that the corresponding strategy profiles are pure Nash-equilibrium profiles. Knowing from proposition 1 that the Nash-equilibrium is unique, we conclude that only one equilibrium with serially undominated strategies can exist. \square

Proof of Proposition 2

First we turn to the lender. In both cases the lenders' payoff expected at the beginning of the second stage (conditional on effort having been provided in the first period) is given as

$$\begin{aligned} \pi_L &= (\hat{x} - \epsilon - \underline{\theta})K + \int_{\hat{x} - \epsilon}^{\hat{\theta}} T(\theta, \hat{x})K d\theta \\ &\quad + \int_{\hat{\theta}}^{\hat{x} + \epsilon} [T(\theta, \hat{x})K + (1 - T(\theta, \hat{x}))D] d\theta \\ &\quad + (\bar{\theta} - \hat{x} - \epsilon)D \\ &= K(\hat{x} - \underline{\theta}) + D(\bar{\theta} - \hat{x}) - \frac{D(\epsilon + \hat{\theta} - \hat{x})^2}{4\epsilon}. \end{aligned}$$

Using (7) this may be rewritten as:

$$\pi_L(\hat{\theta}) = D\bar{\theta} - K\underline{\theta} - (D - K) \left(\frac{\epsilon K}{D} + \hat{\theta} \right).$$

Comparing the profits with committed single lender and uncoordinated multiple lenders yields

$$\pi_L(\theta_0) - \pi_L(\hat{\theta}^*) = (D - K)(\hat{\theta}^* - \theta_0) = \frac{K(W - D)(D - K)}{D} > 0.$$

Now, we turn to the firm. With multiple lenders profit is

$$\begin{aligned}\pi_F^M &= (\hat{\theta}^* - \underline{\theta})V \\ &\quad + \int_{\hat{\theta}^*}^{\hat{x}(\hat{\theta}^*)+\epsilon} [\theta - D - T(\theta, \hat{x})(W - D)] d\theta \\ &\quad + \int_{\hat{x}(\hat{\theta}^*)+\epsilon}^{\bar{\theta}} (\theta - D) d\theta.\end{aligned}$$

With a single committed lender profit is

$$\begin{aligned}\pi_F^S &= (\theta_0 - \underline{\theta})V \\ &\quad + \int_{\theta_0}^{\hat{x}(\theta_0)+\epsilon} [T(\theta, \hat{x})V + (1 - T(\theta, \hat{x}))(\theta - D)] d\theta \\ &\quad + \int_{\hat{x}(\theta_0)+\epsilon}^{\bar{\theta}} (\theta - D) d\theta.\end{aligned}$$

Let I^M and I^S denote the integrand of the middle rows. Then

$$\begin{aligned}I^S - I^M &= -T(\theta, \hat{x}(\theta_0))(\theta - D - V) + T(\theta, \hat{x}(\hat{\theta}^*))(W - D) \\ &> -(\theta - D - V) + (W - D) \\ &> 0,\end{aligned}$$

where the first inequality is due to the shift and the second follows from $\theta < \theta_1 = V + W$. Then $V < I^M < I^S < (\theta - D)$ and $\theta_0 < \hat{\theta}^*$ imply that $\pi_F^S - \pi_F^M > 0$.

Proof of Proposition 3

Being a probability, $p(\theta_0|x) = (V + D - x + \epsilon)/(2\epsilon)$ takes values in $[0, 1]$. For $x > V + W + \epsilon$, the firm will never default even when the old lender charges the maximal amount W . For $x \leq V + W + \epsilon$ we have to account for an interior solution with $p > 0$ and a border solution with $p = 0$. (Obviously $p = 1$ cannot be optimal). In the first case, $\partial R/\partial D = 0$ yields the optimal charge as $D = (x + \epsilon - V)/2$. With this schedule p drops to zero for $x = V + 3\epsilon$. Hence, for $x \in [V + 3\epsilon, V + W + \epsilon]$ the optimal D is derived from $p(\theta_0|x) = 0$ as $D = x - \epsilon - V$. Collecting these results we obtain the optimal charge conditional on refinancing as

$$D^* = \begin{cases} (x + \epsilon - V)/2 & \text{for } x \leq 3\epsilon + V \\ x - \epsilon - V & \text{for } 3\epsilon + V < x < W + V + \epsilon \\ W & \text{for } W + V + \epsilon \leq x \end{cases}$$

For $2\epsilon < K$, the lender prefers to foreclose in the lowest range, thus the switching strategy $\hat{x}^R \in [3\epsilon + V, W + V + \epsilon]$ is obtained from $K = x - \epsilon - V$. \square

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